Electric transmission lines: Control and numerical discretization

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We are interested in the mathematical modeling and control of electrical power lines. The electric transmission lines under consideration are modeled as directed graph. The evolution of voltage and current is described by a linear 2×2 system of hyperbolic balance laws. Open and closed loop control concepts are investigated for control actions at nodal points in the network. For the modeling of electric transmission lines several mathematical approaches can be found in the literature [1, 3]. A typical control problem is to find the optimal power flow when allowing for power supply at selected nodes. Mathematically, this leads to possibly large scale finite-dimensional nonlinear optimization problem [4]. From those, simplified models are derived, which use linear or mixed-integer programming techniques to determine the optimal flow [7]. Based on those models extensions to oscillator networks are used to capture some effects of time-dependent transmission lines. A typical model for the evolution is given by the spatially one-dimensional telegraph equation supplemented by suitable source terms. For instance, in [9] the power and signal losses in communication systems are determined by the telegraph equation. Depending on the sheeted material sometimes lossless transmission lines may be considered. In this case the telegraph equation reduces to the linear wave equation for the voltage. Mathematically, closed loop control questions for the wave equation have extensively been studied for example in [10]. Closed loop control of a single transmission line including source terms has been established using a Lyapunov function approach. We extend the existing result to more general networks and introduce coupling conditions for the telegraph equation as well. The idea is to derive a closed loop network control relying heavily on the construction of the suitable Lyapunov function for the telegraph equation. Furthermore, we introduce an open loop control concept and numerically compare the arising control actions. This extends recent investigations conducted for other applications [3, 11, 12].

References

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