Integration of high index DAEs using the software package QUALIDAES

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We are interested in the numerical integration of *differential-algebraic equations* in *quasi-linear* form

$$M(y,t)\dot{y} = k(y,u,t) \tag{1}$$

where y represents the *unknown variables* and u the (possible) *control*. If we are interested in the determination of a control u forcing the dynamical system into a (partially) prescribed motion or to satisfy path constraints, i.e.,

$$0 \quad = \quad c(y,t), \tag{2}$$

then the resulting problem has the form of a quasi-linear DAE of the form

$$E(x,t)\dot{x} = f(x,t) \tag{3}$$

with $x^T = [y^T \; u^T]$ as unknown variables. Here the d-index can increase arbitrarily and the DAE contains hidden constraints which are, roughly speaking, deeply hidden in the system. This deep hiddenness of constraints complicates the numerical integration extremely.
Due to the hidden constraints which lead to instabilities, convergence problems, or inconsistencies in the direct numerical integration of the equations of motion, a regularization which

R1) reduces the index and
R2) preserves the set of solutions

is necessary.
In this talk we will discuss the efficient and robust numerical integration of quasi-linear DAEs (3) of high index. We will propose an approach which combines a regularization with an eficient numerical integration.
The regularization approach is based on overdetermined formulations where the hidden constraints or derivatives of (parts of) the DAE are added to the DAE. We obtain an overdetermined formulation which corresponds to a regularization satisfying R1) and R2) and is uniquely solvable if the original DAE was uniquely solvable. Therefore, this formulation is suitable for the direct numerical treatment with adapted discretization methods.
Based on that regularization approach we present the software package `QUALIDAES`. This software package is suited for the direct numerical integration of the proposed regularized formulation and uses adapted discretization methods based on the Runge-Kutta method of type RADAU IIa of order 5.
The efficiency and applicability of the proposed approach for the numerical treatment of quasi-linear DAEs (3) of high index will be demonstrated on several examples. Furthermore, a comparison to other widely used solvers like `RADAU5` and `DASSL/DASPK` will be provided.