Discrete Input/Output Maps and a Generalization of the Proper Orthogonal Decomposition Method

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Current control design techniques require system models of moderate size to be applicable. The generation of such models is challenging for complex systems which are typically described by partial differential equations (PDEs), and model-order reduction or low-order-modeling techniques have been developed for this purpose. Many of them heavily rely on the state space models and their discretizations. However, in control applications, a sufficient accuracy of the models with respect to their input/output (I/O) behavior is typically more relevant than the accurate representation of the system states.

In this talk, we present a discretization framework which has been developed recently [1] and which heavily focuses on the I/O map of the original PDE system. In particular, the proposed direct discretization of the I/O map of a linear time-invariant system comes with error bounds measuring the relevant I/O error. We show how the discretized I/O map can be realized as a matrix. By tensor techniques the I/O matrix can be further reduced to a very low-dimensional map which is shown to be beneficial in a control application.

For special choices of input and output spaces, the proposed reduction coincides with the well-known Proper Orthogonal Decomposition (POD) method. Turning this argument around, we find that the method of discretizing I/O maps can be employed for a generalization of the common POD method. We present numerical examples [2] that demonstrate the benefits of generalized POD.

References
