On the Comparison of Sufficient Conditions for the Real and Symmetric Nonnegative Inverse Eigenvalue Problems

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The real nonnegative inverse eigenvalue problem (RNIEP) is the problem of characterizing all possible real spectra of entrywise nonnegative matrices. This problem remains unsolved. Since the first result in this area announced by Suleimanova in 1949 and proved by Perfect in 1953, a number of realizability criteria or sufficient conditions for the existence of a nonnegative matrix with a given real spectrum have been obtained, from different points of view. In [2] the authors construct a map of sufficient conditions for the RNIEP, in which they show inclusion or independency relations between these conditions.
If in the RNIEP we require that the nonnegative matrix be symmetric, we have the symmetric nonnegative inverse eigenvalue problem (SNIEP). The first known sufficient condition for the SNIEP is due to Perfect and Mirsky in 1965 for doubly stochastic matrices, and Fiedler gave in 1974 the first symmetric realizability criteria for nonnegative matrices. It is well known that these two problems are equivalent for spectra of size $n \leq 4$ and a complete solution of both is known only for $n \leq 4$. For $n \geq 5$ they are different and both problems remain open.
Given a real spectrum σ verifying a sufficient condition X, we introduce the X-margin of realizability of σ to measure how much we can decrease the spectral radius of σ preserving the sufficient condition X. We analyze several sufficient conditions from the point of view of their margin of realizability [1].
Since 2007 new sufficient conditions for the RNIEP have appeared. We discuss new relations of inclusion or independency between these new sufficient conditions and the previous ones studied in [3]. We also construct a map of sufficient conditions for the SNIEP [4]. Finally, we describe and discuss some open problems of interest in this context.

References

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