On the backward stability of computing polynomial roots via colleague matrices

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Computing the roots of scalar and matrix polynomials expressed in the Chebyshev basis \(\{T_k(x)\}\) is a fundamental problem that arises in many applications. For instance, a standard way to compute the real roots of a smooth function \(f(x)\) on an interval is to approximate \(f(x)\) by a polynomial \(p(x)\) via Chebyshev interpolation. A common way of computing the roots of a polynomial expressed in the Chebyshev basis is to compute the eigenvalues of its colleague matrix. In this work, we analyze the backward stability of the polynomial root-finding problem solved with colleague matrices. In other words, given the polynomial \(P(x) = T_n(x) + \sum_{k=0}^{n-1} A_k T_k(x)\), with \(A_k \in \mathbb{R}^{p \times p}\), expressed in the Chebyshev basis, the question is to determine whether the whole set of computed eigenvalues of the colleague matrix, obtained with a backward stable algorithm like the QR-algorithm for the standard eigenvalue problem, are the set of roots of a nearby polynomial or not. This question was answer by A. Edelman and H. Murakami in [1] when the polynomial is expressed in the monomial basis. In this work, we derive a first order backward error analysis of the polynomial root-finding algorithm using colleague matrices following a different approach to the one followed by A. Edelman and H. Murakami. Our backward error analysis expands on a very recent work by Y. Nakatsukasa and V. Noferini [2] in that we show that this algorithm is backward normwise stable if the coefficients of the polynomial \(P(x)\) have moderate norms. We also present numerical experiments that support these theoretical results.

References
