Structured eigenvalue backward errors of matrix pencils

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An $n \times n$ regular matrix pencil $L(z) = zA + B$ is said to be structured if (A, B) belong to a special subset of $(\mathbb{C}^{n\times n})^2$. Matrix pencils arising in most applications follow some structure and the use of structure-preserving algorithms is advisable for solving them. Structure preserving perturbation analysis is necessary to assess the accuracy of such algorithms. This involves computing backward errors with respect to structure preserving perturbations which we refer to as structured backward errors.

The structured eigenvalue backward errors are important for the stability analysis of structure preserving algorithms that compute only eigenvalues and for solving distance problems involving structured matrices [4]. Explicit formulas for structured eigenpair backward errors of matrix pencils have been developed in [1, 2, 3] for a number of important structures. However, structured eigenvalue backward errors have not been obtained in these works. Motivated by such considerations, explicit formulas for structured eigenvalue backward errors have been obtained for matrix pencils with Hermitian and related structures in [5] and for palindromic structures in [6] with respect to the norm $\sqrt{\|A\|_2^2 + \|B\|_2^2}$ on $L(z)$, where $\|\cdot\|_2$ is the matrix 2-norm.

In this talk, we briefly present some of the main results in [5] and [6], and focus on extensions to the case where the norm on $L(z)$ is $\max\{\|A\|_2, \|B\|_2\}$. An important structure that often arises in applications is when the coefficient matrices of the pencil $L(z)$ are real. Understanding the effect of real perturbations on real matrix pencils is a challenging task. The real eigenvalue and eigenpair backward errors are not known for real matrix pencils even when they have no additional structure. We present some results for structured eigenvalue/eigenpair backward errors of real structured pencils with respect to perturbations that preserve realness as well as additional symmetries. All the results are illustrated by numerical experiments.

References

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