Linearization schemes for Hermite matrix polynomials

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The polynomial eigenvalue problem is to find the eigenpair of $(\lambda, x) \in \mathbb{C}\bigcup\{\infty\} \times \mathbb{C}^n\backslash\{0\}$ that satisfies $P(\lambda)x = 0$, where $P(\lambda) = \sum_{i=0}^{s} P_i\lambda^i$ is an $n \times n$ so-called matrix polynomial of degree s, where the coefficients $P_i, i = 0, \cdots, s$, are $n \times n$ constant matrices, and P_s is supposed to be nonzero. These eigenvalue problems arise from a variety of physical applications including acoustic structural coupled systems, fluid mechanics, multiple input multiple output systems in control theory, signal processing, and constrained least square problems. Most numerical approaches to solving such eigenvalue problems proceed by linearizing the matrix polynomial into a matrix pencil of larger size.

Such methods convert the eigenvalue problem into a well-studied linear eigenvalue problem, and meanwhile, exploit and preserve the structure and properties of the original eigenvalue problem. The linearizations have been extensively studied with respect to the basis that the matrix polynomial is expressed in. If the matrix polynomial is expressed in a special basis, then it is desirable that its linearization be also expressed in the same basis. The reason is due to the fact that changing the given basis ought to be avoided [3]. The authors in [1] have constructed linearization for different bases such as degree-graded ones (including monomial, Newton and Pochhammer basis), Bernstein and Lagrange basis. This contribution is concerned with polynomial eigenvalue problems in which the matrix polynomial is expressed in Hermite basis. In fact, Hermite basis is used for presenting matrix polynomials designed for matching a series of points and function derivatives at the prescribed nodes.

In the literature, the linearizations of matrix polynomials of degree s, expressed in Hermite basis, consist of matrix pencils with $s + 2$ blocks of size $n \times n$. In other words, additional eigenvalues at infinity had to be introduced, see e.g. [2]. In this research, we try to overcome this difficulty by reducing the size of linearization. The reduction scheme presented will gradually reduce the linearization to its minimal size making use of ideas from [4]. More precisely, for $n \times n$ matrix polynomials of degree s, we present linearizations of smaller size, consisting of $s+1$ and s blocks of $n \times n$ matrices. The structure of the eigenvectors is also discussed.

References

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