Inexact nested Newton-ADI method to solve large-scale algebraic Riccati equations

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We investigate numerical methods to efficiently solve algebraic Riccati equations (ARE) like

$$\mathcal{R}(X) = C^T C + A^T X + XA - XBB^T X = 0 \tag{1}$$

with $C \in \mathbb{R}^{p\times n}$, $A \in \mathbb{R}^{n\times n}$, $X = X^T \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $p + r \ll n$, by combing existing approaches. These quadratic matrix equations have to be solved, e.g., in optimal control problems to apply a *linear quadratic regulator* (LQR) approach [6, 7].
Our aim is an iterative solver for (1) based on the Newton-ADI method. Recent ADI improvements in [2, 3, 4] in combination with the inexact Kleinman-Newton approach in [5] and the line search method in [1] are the key ingredients to our novel approach that can handle large-scale problems efficiently.
We show theoretical as well as numerical results that illustrate the usability of the novel approach as well as its advantages.
The open problem of controlling the accuracy of the solver for the shifted linear systems appearing in each ADI step will also be addressed shortly.

References

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