



Conference in Honor of Volker Mehrmann on the Occasion  
of his 60th Birthday

$$E(x, t) \dot{x} = f(x, t) + D(x) \Delta_{-2} x$$

$$\begin{bmatrix} L & U \\ -I & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} E \\ F^T \end{bmatrix}$$

# Numerical Algebra, Matrix Theory, Differential-Algebraic Equations, and Control Theory

$$\dot{x} = Fx + f(t)$$

$$x \geq 0 \quad t \in \mathbb{R}$$

$F(t, x, x)$  not strong regularity

↓ Hypothesis

$F_1(t, x, x)$  strongness free

$F_2(t, x)$  KM-regularization

$$E \dot{x} = Fx + Bu$$

$$y = Cx$$

$$\Rightarrow G(\omega) = C(\omega E - F)^{-1} B$$

$$(E - 2tF) v = 0$$

$$\begin{bmatrix} A^T & B-A \\ A^T & A^T \end{bmatrix} + \begin{bmatrix} A & A \\ B-A & A \end{bmatrix}$$

May 6-9, 2015

Berlin, Germany

**Program & Book of Abstracts**

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## Scientific committee and organizers:

Peter Benner	(MPI for Dynamics of Complex Technical Systems, Magdeburg)
Jörg Liesen	(TU Berlin)
Christian Mehl	(TU Berlin)
Reinhard Nabben	(TU Berlin)
Lena Scholz	(TU Berlin)
Andreas Steinbrecher	(TU Berlin)

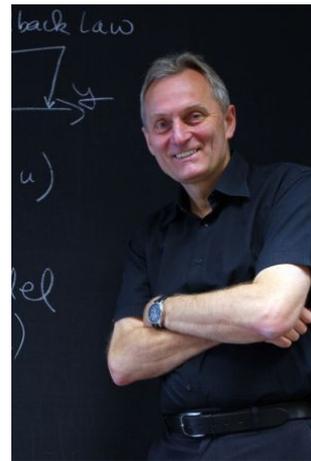


## **General information**

## Numerical Algebra, Matrix Theory, Differential-Algebraic Equations, and Control Theory

This conference aims at bringing together experts in the fields of numerical (linear) algebra, matrix theory, differential-algebraic equations and control theory. These mathematical research areas are strongly related and they often occur in the same real-world application. Main areas where such applications emerge are computational engineering and sciences, but increasingly also social sciences and economics.

The conference is dedicated to Volker Mehrmann on the occasion of his 60th birthday. Volker Mehrmann is a leading expert in the areas of the conference, and in a unique manner unifies expertise in the mathematical fields providing the title of this conference.



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### Web page and information email

Conference web page: <http://www3.math.tu-berlin.de/multiphysics/VM60/>

Conference email: [vm60info@math.tu-berlin.de](mailto:vm60info@math.tu-berlin.de)

### Location



TU Berlin  
Institut für Mathematik  
Straße des 17. Juni 136  
10623 Berlin, Germany

### Presentation preparation for talks and posters

#### *Oral presentations:*

The conference room will be equipped with a computer running MS Windows with MS Office and Adobe Acrobat Reader. A wireless presenter will be available. Please upload your presentation as soon as possible, at the latest in the break before the scheduled talk. Speakers may use their own laptops if they wish. In any case, please check your hardware and presentation in advance.

#### *Poster presentations:*

Presentation boards will be available from Thursday morning beginning at 8:00 on the first floor along the gallery, where also the coffee breaks will take place. Please use the board that has a printout of your abstract attached to it. For fixing the poster, power strips will be available at the registration desk. Please attach your poster to the presentation board as soon as possible, but at the latest in the break preceding the poster session.

## Internet access

Eduroam is available via WiFi in the Math building and on the campus of the TU Berlin. Guest accounts for WiFi can be provided. For more information please ask the staff at the registration desk.

## Coffee breaks, lunch

The *coffee breaks* take place on the first floor of the Math building outside of the conference room MA001 and along the gallery.

*Lunch options:*

- in the "Kantine" on the 9th floor of the Math building (cash only),
- in the cafeteria near the conference room on the ground floor of the Math building (cash only),
- in the "Café Campus" behind the Math building (see map on page 86, cash only),
- in the "Mensa" (Hardenbergstraße 34, see map on page 86, only with "MensaCard" (prepaid card) available in the Mensa, prepay with cash only), or
- in the Knesebeckstraße (see map on page 86), where you can find several restaurants.

## Welcome reception & conference dinner

*Welcome reception:*

**"Lichthof", Main Building of the TU Berlin (Straße des 17. Juni 135, 10623 Berlin)  
on Wednesday, May 6 at 18:30.**

The "Lichthof" is located on the first floor of the main building of the TU Berlin, marked as "Hauptgebäude" on the map on page 86. The main building is located across the street from the main entrance of the Math building.

*Conference dinner:*

**Restaurant "Alte Pumpe" (Lützowstraße 42, 10785 Berlin)  
on Friday, May 8 at 19:30.**

A buffet will be offered with several courses including vegetarian meals. A selection of drinks will also be included from 19:30 until 23:30. The dinner will start with a Berlin-style *Currywurst reception*.

**Please do not forget your voucher.**

The restaurant "Alte Pumpe" is about 30 minutes away from the Math building, either by foot or public transportation (see next pages for maps and directions). Below is a picture of the street entrance to "Alte Pumpe".

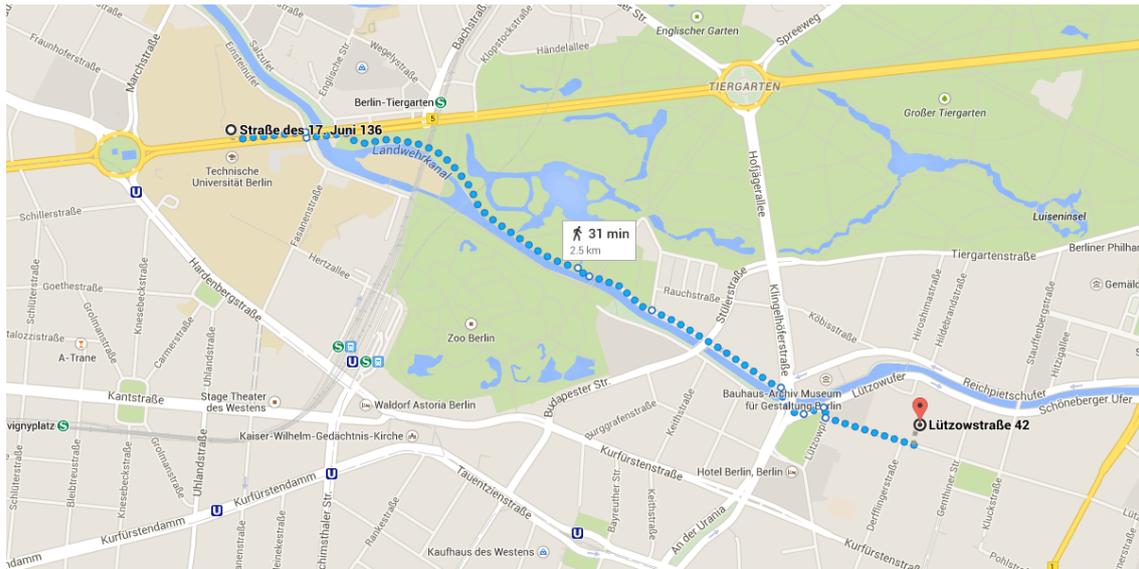


Entrance to the Restaurant

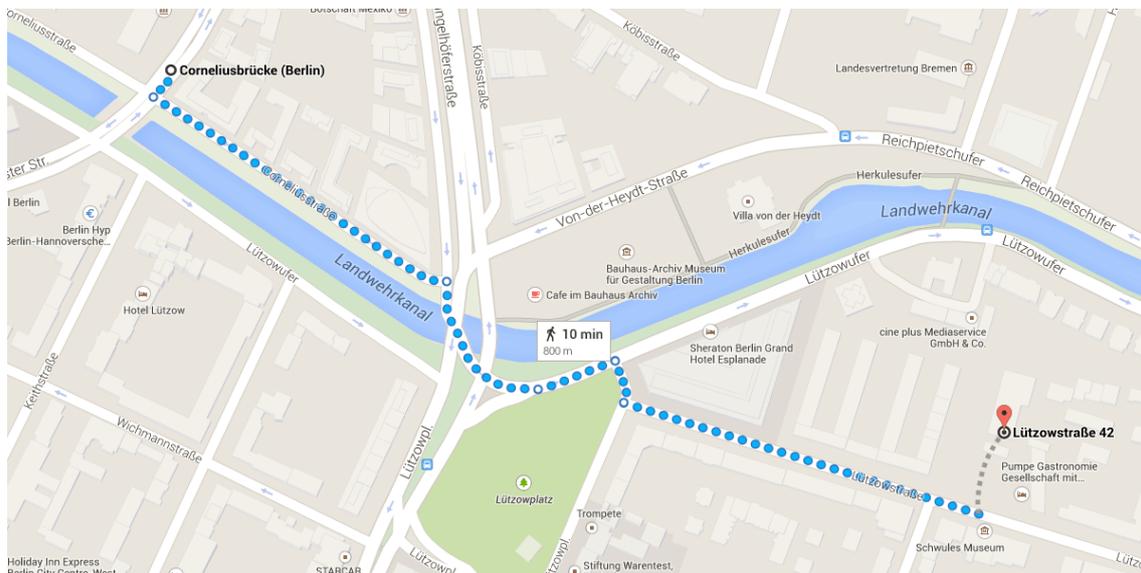
### Option 1: Walking from the Math building (2.6km)

The route is shown in the maps below. The directions are as follows:

- Head east on “Straße des 17. Juni” towards the “Tiergarten” park.
- On “Tiergartenufer”, turn right and keep walking for about 1.2km with the “Landwehrkanal” on your right.
- Exit the park and continue on “Corneliusstraße”.
- Cross the canal, continue through “Lützowufer” to “Lützowstraße”.
- Enter the restaurant through the children playgrounds located at “Lützowstraße” 42.



Map of walking route to the restaurant where the conference dinner will take place.



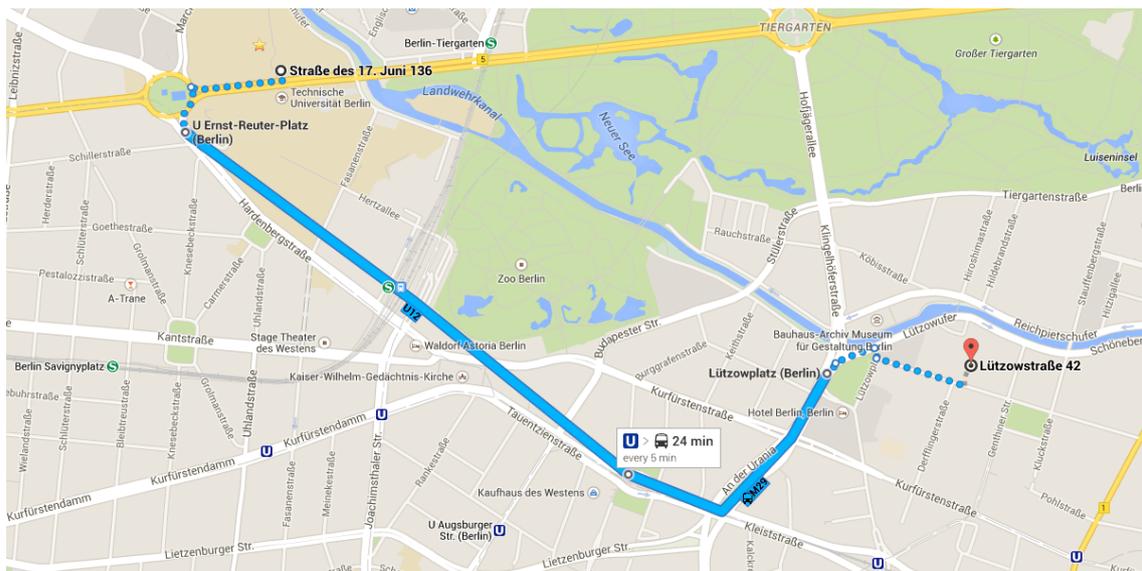
Close-up of “Lützowplatz”.

## Option 2: Subway & Bus ride from the Math building or Zoologischer Garten station

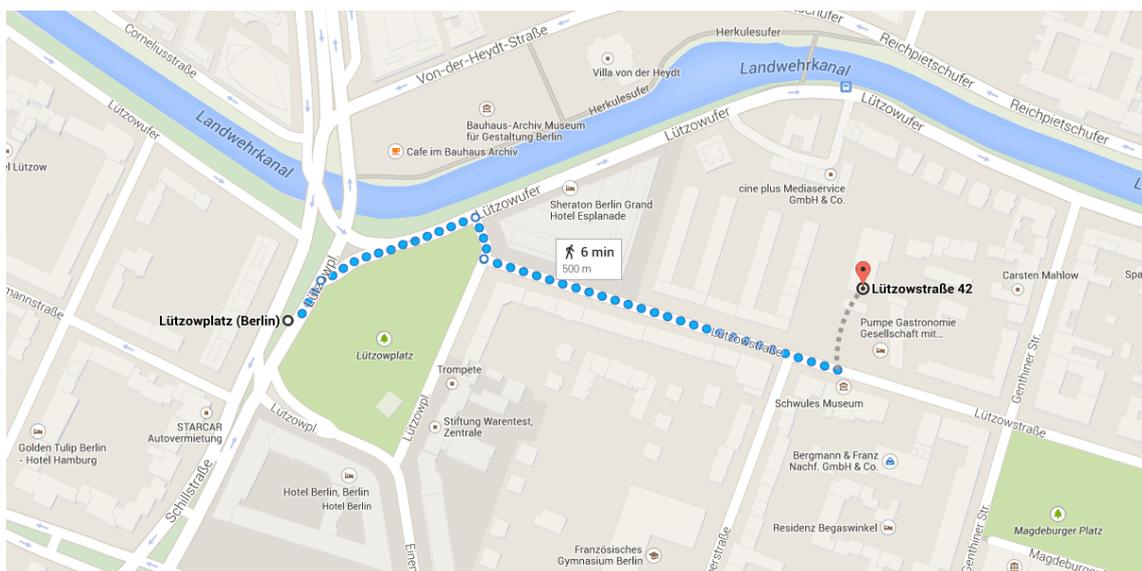
The route is shown in the maps below. The directions are as follows:

- From the Math building, head west on “Straße des 17. Juni” and go to the subway station “**U** Ernst-Reuter-Platz”.
- Take the subway line **U12** with direction to “Warschauer Straße” and leave the subway after two stops at the station “Wittenbergplatz”.
- Take the bus **M45** with direction “U Hermannplatz” and get off at the stop “Lützowplatz”.
- Walk to “Lützowstraße” and enter the restaurant through the children playgrounds located at “Lützowstraße” 42.

If you start from the “Zoologischer Garten” station, take the line **U12** with direction to “Warschauer Straße”, get off at “Wittenbergplatz” and follow the directions above.



Map of public transport route to the restaurant where the conference dinner will take place.



Close-up of “Lützowplatz”



# Schedule

## Wednesday, May 6

Time	Event / Talk	Room	Abstract on page
08:00-09:00	Registration		
09:00-09:30	<b>Opening</b> Prof. Dr. Christian Thomsen (President of the TU Berlin) Prof. Dr. Christof Schütte (Vice-President Zuse Institute Berlin and Co-Chair DFG Research Center MATHEON) Prof. Dr. Peter Benner (Director Max Planck Institute for Dynamics of Complex Technical Systems)	MA001	
	<b>INVITED TALKS</b> Session Chair: Jörg Liesen		
09:30-10:00	Maria J. Esteban: <i>Symmetry and symmetry breaking for optimizers of functional inequalities</i>	MA001	18
10:00-11:00	Coffee break		
	<b>INVITED TALKS</b> Session Chair: Angelika Bunse-Gerstner		
11:00-11:30	Shreemayee Bora: <i>Structure preserving perturbations and distance problems: a decade of work with Volker Mehrmann</i>	MA001	16
11:30-12:00	Heike Faßbender: <i>Complex <math>J</math>-symmetric eigenproblems – more on structure-preserving algorithms for structured eigenvalue problems</i>	MA001	19
12:00-12:30	Julio Moro: <i>Structured vs. unstructured spectral perturbation: a particular overview</i>	MA001	24
12:30-14:00	Lunch		
	<b>CONTRIBUTED TALKS</b> Session Chair: Heike Faßbender		
14:00-14:20	Rafikul Alam: <i>Fiedler-like pencils for rational eigenvalue problems</i>	MA001	35
14:20-14:40	Jan Heiland: <i>Discrete input/output maps and a generalization of the proper orthogonal decomposition method</i>	MA001	43
14:40-15:00	Marc Van Barel: <i>Generalization of Lagrange linearization for polynomial eigenvalue problems</i>	MA001	48
15:00-15:20	Thanos Antoulas: <i>Data-driven model reduction in the Loewner framework</i>	MA001	36
15:20-15:40	Thomas Berger: <i>On the regularization of linear time-invariant descriptor systems</i>	MA001	37
15:40-16:30	Coffee break		
	<b>INVITED TALKS</b> Session Chair: David Watkins		
16:30-17:00	Daniel Szyld: <i>Asynchronous optimized Schwarz methods</i>	MA001	28
17:00-17:30	Valeria Simoncini: <i>Decay pattern of matrices: application to matrix functions and matrix equations</i>	MA001	26
	<b>CONTRIBUTED TALKS</b> Session Chair: David Watkins		
17:30-17:50	Froilán M. Dopico: <i>Volker Mehrmann and modern factorizations of symplectic matrices</i>	MA001	42
17:50-18:10	Thomas Laffey: <i>Estimating the traces of powers of certain nonnegative matrices related to orthogonal polynomials</i>	MA001	44
18:30-20:00	<b>Welcome reception</b>	Lichthof	

**Thursday, May 7**

Time	Event / Talk	Room	Abstract on page
08:00-09:00	Registration		
	<b>INVITED TALKS</b>	Session Chair: Stephen Campbell	
09:00-9:30	Caren Tischendorf: <i>Modeling and numerical analysis of PDAEs describing flow networks</i>	MA001	29
9:30-10:00	Peter Kunkel: <i>On the geometric integration of self-adjoint linear DAEs</i>	MA001	21
10:00-10:30	Achim Ilchmann: <i>Behaviour of time-varying DAEs</i>	MA001	21
10:30-11:00	Coffee break		
	<b>INVITED TALKS</b>	Session Chair: Timo Reis	
11:00-11:30	Vu H. Linh: <i>Spectrum-based robust stability analysis of linear delay differential-algebraic equations</i>	MA001	22
11:30-12:00	Tatjana Stykel: <i>Model reduction of linear and nonlinear magneto-quasistatic equations</i>	MA001	27
12:00-12:30	Etienne Emmrich: <i>The peridynamic model in nonlocal elasticity theory</i>	MA001	17
12:30-14:00	Lunch		
	<b>CONTRIBUTED TALKS</b>	Session Chair: Tatjana Stykel	
14:00-14:20	Steffen Börm: <i>Hierarchical vectors</i>	MA001	38
14:20-14:40	Eugene Tyrtshnikov: <i>Rank structures in multidimensional matrices</i>	MA001	48
14:40-15:00	Fernando De Terán: <i>Low rank perturbation of canonical forms</i>	MA001	40
15:00-15:20	André Ran: <i>Eigenvalues of rank one perturbations of matrices with structure in an indefinite inner product space</i>	MA001	46
15:20-15:40	Tobias Damm: <i>Dual pairs of Lyapunov inequalities</i>	MA001	39
15:40-16:30	Coffee break		
	<b>INVITED TALKS</b>	Session Chair: Christian Mehl	
16:30-17:00	Paul Van Dooren: <i>Structured backward stability of linearizations of polynomial matrices</i>	MA001	49
	<b>POSTER BLITZ AND POSTER SESSION</b>	Session Chair: Christian Mehl	
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Nieves Castro-González: <i>Perturbation theory of the Moore-Penrose inverse and the least squares problem</i>	53
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## Friday, May 8

Time	Event / Talk	Room	Abstract on page
08:00-09:00	Registration		
	<b>INVITED TALKS</b>	Session Chair: Françoise Tisseur	
09:00-09:30	Timo Reis: <i>On ADI approximate balanced truncation</i>	MA001	26
09:30-10:00	Agnieszka Międlar: <i>Moving eigenvalues and eigenvectors - perturbation theory in practice</i>	MA001	24
10:00-10:30	Matthias Bollhöfer: <i>Algorithms for computing functions of matrix inverses</i>	MA001	15
10:30-11:00	Coffee break		
	<b>INVITED TALKS</b>	Session Chair: Valeria Simoncini	
11:00-11:30	David Watkins: <i>Fast and backward stable computation of the zeros of polynomials</i>	MA001	31
11:30-12:00	Hongguo Xu: <i>Compressing the coefficient matrices of a singular matrix polynomial</i>	MA001	32
12:00-12:30	D. Steven Mackey: <i>Matrix polynomials in non-standard form</i>	MA001	23
12:30-14:00	Lunch		
	<b>CONTRIBUTED TALKS</b>	Session Chair: Daniel Szyld	
14:00-14:20	Andrii Dmytryshyn: <i>Orbit closure hierarchies of skew-symmetric matrix pencils</i>	MA001	41
14:20-14:40	Zdeněk Strakoš: <i>On the Vorobyev method of moments</i>	MA001	47
14:40-15:00	Lothar Reichel: <i>A generalized Krylov subspace method for <math>\ell_p</math>-<math>\ell_q</math> minimization</i>	MA001	47
15:00-15:20	Javier Pérez: <i>On the backward stability of computing polynomial roots via colleague matrices</i>	MA001	45
15:20-16:00	Coffee break		
	<b>A SPECIAL SESSION DEDICATED TO VOLKER MEHRMANN</b>	Session Chair: Reinhard Nabben	
16:00-18:00	Angelika Bunse-Gerstner, Nancy Nichols, Fredi Tröltzsch, Peter Benner, Christian Mehl		
19:30	Conference dinner at restaurant "Alte Pumpe"		

**Saturday, May 9**

Time	Event / Talk	Room	Abstract on page
09:00-10:00	Registration		
	<b>INVITED TALKS</b>		
		Session Chair: Andreas Steinbrecher	
10:00-10:30	Martin Grötschel: <i>Numerical linear algebra: a view from an outsider</i>	MA001	20
10:30-11:00	Stephen Campbell: <i>Some advantages of a DAE formulation</i>	MA001	17
11:00-11:30	Coffee break		
	<b>INVITED TALKS</b>		
		Session Chair: Peter Benner	
11:30-12:00	Federico Poloni: <i>Cyclic reduction and index reduction/shifting for a second-order probabilistic problem</i>	MA001	25
12:00-12:30	Shmuel Friedland: <i>Low rank approximation of tensors</i>	MA001	20
12:30-13:00	Françoise Tisseur: <i>Max-Balancing Hungarian Scalings</i>	MA001	29
13:00-13:30	<b>Closing</b>	MA001	

## **Abstracts of invited talks**



## Algorithms for computing functions of matrix inverses

M. Bollhöfer<sup>1</sup>

<sup>1</sup>TU Braunschweig, Institute for Computational Mathematics, m.bollhoefer@tu-bs.de

Functions of entries of inverses of matrices like all diagonal entries of a sparse matrix inverse or its trace arise in several important computational applications such as density functional theory [2], covariance matrix analysis in uncertainty quantification [1], vehicle acoustics optimization [6], or when evaluating Green's functions in computational nanoelectronics [4]. We will review some methods for (approximately) computing selective parts of the matrix inverse such as stochastic estimators [1], domain decomposition-based methods [7, 5] or direct methods [3]. We will further present a new algorithm for approximate selective matrix inversion that uses an approximate version of the method presented in [3]. Its overall performance will be demonstrated for selected numerical examples, in particular for symmetric and indefinite application problems which frequently arise from practical applications.

### References

- [1] C. Bekas, A. Curioni, and I. Fedulova. Low-cost high performance uncertainty quantification. *Concurrency and Computation: Practice and Experience*, 2011.
- [2] W. Kohn, L. Sham, et al. Self-consistent equations including exchange and correlation effects. *Phys. Rev*, 140(4A):A1133–A1138, 1965.
- [3] L. Lin, C. Yang, J. C. Meza, J. Lu, L. Ying, and W. E. Sellnv – an algorithm for selected inversion of a sparse symmetric matrix. *ACM Transactions on Mathematical Software*, 37(4):40:1–40:19, 2011.
- [4] M. Luisier, T. Boykin, G. Klimeck, and W. Fichtner. Atomistic nanoelectronic device engineering with sustained performances up to 1.44 pflop/s. In *High Performance Computing, Networking, Storage and Analysis (SC), 2011 International Conference for*, pages 1–11. IEEE, 2011.
- [5] V. Mehrmann. Divide & conquer methods for block tridiagonal systems. *Parallel Comput.*, 19:257–279, 1993.
- [6] V. Mehrmann and C. Schröder. Nonlinear eigenvalue and frequency response problems in industrial practice. *J. Math. in Industry*, 1:7, 2011.
- [7] J. M. Tang and Y. Saad. Domain-decomposition-type methods for computing the diagonal of a matrix inverse. *SIAM J. Sci. Comput.*, 33(5):2823–2847, 2011.

## Structure preserving perturbations and distance problems: a decade of work with Volker Mehrmann

S. Bora<sup>1</sup>

<sup>1</sup>Department of Mathematics, Indian Institute of Technology Guwahati, India, shbora@iitg.ernet.in

Much of Volker Mehrmann's work in control theory and mathematical modelling is associated with solutions of challenging eigenvalue problems. Very often, the challenge is the result of the fact that the matrices involved in the problems have some special structure resulting in eigenvalues that are symmetrically placed with respect to some subset in the complex plane. Eigenvalues belonging to these subsets are called critical eigenvalues as they do not conform to the existing eigenvalue symmetry and their movements are restricted by some additional attributes called sign characteristics. Existence of critical eigenvalues can pose significant computational challenges and undesirable physical phenomena like loss of passivation [1]. Volker Mehrmann is among the early researchers to acknowledge the importance of analysing the effect of structure preserving perturbations on such eigenvalue problems and their role in the solutions of certain 'distance problems' that often arise in applications. This talk will present some of the challenging distance problems that were tackled in [1] and [2] from the point of view of structure preserving perturbation analysis. Similar analysis has been used to solve other classes distance problems in [3] and [4]. A brief overview of these will also be given.

### References

- [1] R. Alam, S. Bora, M. Karow, V. Mehrmann and J. Moro, Perturbation theory for Hamiltonian matrices and the distance to bounded realness, *SIAM J. Matrix Anal. Appl.*, 32(2011), pp. 484-514.
- [2] S. Bora and V. Mehrmann, Linear perturbation theory for structured matrix pencils arising in control theory, *SIAM J. Matrix Anal. Appl.*, 28(2006), pp. 148-191.
- [3] R. Srivastava, *Distance problems for Hermitian matrix pencils and polynomials - an  $\epsilon$  pseudospectra based approach*. PhD Thesis, Department of Mathematics, IIT Guwahati, India, December 2012.
- [4] S. Bora and R. Srivastava, Distance problems for Hermitian matrix pencils with eigenvalues of definite type, 2014. Submitted to *SIAM J. Matrix Anal. Appl.*

## Some advantages of a DAE formulation

S. Campbell<sup>1</sup>

<sup>1</sup>North Carolina State University, slc@ncsu.edu

It is well known that one advantage of a DAE formulation is that it is often the natural way that many physical systems are formulated and thus DAEs permit easier modeling of a variety of complex processes. However DAE formulations and approaches are sometimes better even for problems with ordinary differential equation models. In this talk we will give several examples drawn from the work of the author and several other researchers. The examples are chosen both to provide a variety of applications as well as a variety of different advantages.

Some examples will be from control theory and in particular observer design. We will not discuss the design of observers for DAEs, this is done elsewhere. Rather, we will present two different examples where the flexibility of a DAE formulation when designing observers can be exploited. One is in getting linear error dynamics. The other is in estimating disturbances. A second set of example deals with the numerical solution of optimal control problems with and without delays.

For each example a basic introduction to the problem will be provided so that the talk will hopefully be accessible to a wide audience.

## The peridynamic model in nonlocal elasticity theory

E. Emmrich<sup>1</sup> and D. Puhst<sup>2</sup>

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<sup>2</sup>TU Berlin, puhst@math.tu-berlin.de

Peridynamics is a nonlocal continuum theory which avoids any spatial derivative. It is believed to be suited for the description of fracture and other material failure, and to model multiscale problems. In this talk, we introduce the peridynamic model and discuss several aspects of its mathematical analysis. We review recent results on the existence of solutions to the peridynamic equation of motion for a large class of nonlinear pairwise force functions modeling isotropic microelastic material (see [1, 2]). Our method of proof applies also to other nonlocal evolution equations.

### References

- [1] E. Emmrich and D. Puhst, Measure-valued and weak solutions to the nonlinear peridynamic model in nonlocal elastodynamics. *Nonlinearity* 28 (2015) 1, pp. 285–307.
- [2] E. Emmrich and D. Puhst, Well-posedness of the peridynamic model with Lipschitz continuous pairwise force function. *Commun. Math. Sci.* 11 (2013) 4, pp. 1039–1049.

## Symmetry and symmetry breaking for optimizers of functional inequalities

M. J. Esteban<sup>1</sup>

<sup>1</sup>CEREMADE. CNRS & Université Paris-Dauphine, esteban@ceremade.dauphine.fr

In this talk will be presented a series of results about the symmetry properties of optimizers of functional inequalities which are invariant under a certain symmetry group. The symmetry issue is of big importance in many of the applications of those inequalities, and also in the study of many physical systems for which knowing when the symmetry is broken is of the utmost importance. Also, when numerical simulations have to be made, the knowledge of the symmetry class where to compute can enormously reduce the computational effort.

The results which will be presented during this talk are mainly theoretical, but the results of numerical computations that have been instrumental in building conjectures all along this project will also be shown.

Some of the works presented here have been obtained with one or several of the following collaborators: J. Dolbeault, M. Loss, G. Tarantello and A. Tertikas.

## Complex $J$ -symmetric eigenproblems – more on structure-preserving algorithms for structured eigenvalue problems

P. Benner<sup>1</sup>, H. Faßbender<sup>2</sup>, and C. Yang<sup>3</sup>

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<sup>2</sup>TU Braunschweig, Institut Computational Mathematics, Braunschweig, h.fassbender@tu-braunschweig.de

<sup>3</sup>Lawrence Berkeley National Laboratory, Computational Research Division, Berkeley, USA, cyang@lbl.gov

The eigenproblem  $H_C x = \lambda x$  for matrices

$$H_C = \begin{bmatrix} A & C \\ D & -A^T \end{bmatrix} \in \mathbb{C}^{2n \times 2n}, \quad A, C = C^T, D = D^T \in \mathbb{C}^{n \times n}.$$

will be considered. Please note, that here  $X^T$  denotes transposition,  $Y = X^T, y_{ij} = x_{ji}$ , no matter whether  $X$  is real or complex, while  $X^H$  denotes conjugate transposition,  $Y = X^H, y_{ij} = \overline{x_{ji}}$ .

For

$$J_n = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \quad I_n \in \mathbb{R}^{n \times n}$$

we have

$$(H_C J)^T = H_C J.$$

Matrices  $H_C$  are called complex- $J$ -symmetric. The eigenvalues of  $H_C$  display a symmetry: they appear in pairs  $(\lambda, -\lambda)$ . If  $x$  is the right eigenvector corresponding to  $\lambda$ ,  $H_C x = \lambda x$ , then  $Jx$  is the left eigenvector corresponding to the eigenvalue  $-\lambda$  of  $H_C$ ,  $(Jx)^T H_C = -\lambda(Jx)$ .

Any complex  $J$ -symmetric matrix  $X$  is said to be in structured Schur form if

$$X = \begin{bmatrix} R & B \\ 0 & -R^T \end{bmatrix}, \quad R, B = B^T \in \mathbb{C}^{n \times n},$$

where the nonzero eigenvalues of  $R$  either have positive real part or zero real part and positive imaginary part. For any complex  $J$ -symmetric matrix  $H_C$  there exists a complex symplectic and unitary matrix  $W \in \mathbb{C}^{2n \times 2n}$

$$W^T J W = J \quad W^H W = I,$$

such that  $W^H H_C W$  is in structured Schur form.

The most popular way to compute the standard Schur form of a general matrix is the  $QR$  algorithm. It is tempting to derive a structured QR algorithm for transforming  $H_C$  iteratively into structured Schur form. We will discuss why this is not possible in general and suggest other methods to compute eigenvalues and eigenvectors of  $H_C$ . In particular, a straightforward adaption of the algorithm for computing the real SR decomposition as given in [1] gives an algorithm for computing the complex symplectic SR decomposition of an arbitrary matrix  $A \in \mathbb{C}^{2n \times 2n}$ . Adapting this complex SR algorithm for complex  $J$ -symmetric  $H_C$  only  $\mathcal{O}(n)$  flops per SR step are needed compared to  $\mathcal{O}(n^3)$  flops when working on a general complex matrix.

Throughout this talk connections to Volker's work on Hamiltonian eigenvalue problems will be highlighted.

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## Low rank approximation of tensors

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In many applications such as data compression, imaging or genomic data analysis, it is important to approximate a given tensor by a tensor that is sparsely representable. For matrices, i.e. 2-tensors, such a representation can be obtained via the singular value decomposition, which allows to compute best rank  $k$ -approximations. For very big matrices a low rank approximation using SVD is not computationally feasible. In this case different approximations are available. It seems that variants of the CUR-decomposition are most suitable.

For  $d$ -mode tensors  $T \in \otimes_{i=1}^d \mathbb{R}^{n_i}$ , with  $d > 2$ , many generalizations of the singular value decomposition have been proposed to obtain low tensor rank decompositions. The most appropriate approximation seems to be best  $(r_1, \dots, r_d)$ -approximation, which maximizes the  $\ell_2$  norm of the projection of  $T$  on  $\otimes_{i=1}^d \mathbf{U}_i$ , where  $\mathbf{U}_i$  is an  $r_i$ -dimensional subspace  $\mathbb{R}^{n_i}$ . One of the most common methods is the *alternating maximization method* (AMM). It is obtained by maximizing on one subspace  $\mathbf{U}_i$ , while keeping all other fixed, and alternating the procedure repeatedly for  $i = 1, \dots, d$ . Usually, AMM will converge to a local best approximation. This approximation is a fixed point of a corresponding map on Grassmannians. We suggest a Newton method for finding the corresponding fixed point. We also discuss variants of CUR-approximation method for tensors. We compare numerically different approximation methods.

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## Numerical linear algebra: a view from an outsider

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The definition that numerical linear algebra is the investigation of algorithms for performing linear algebra computations, in particular matrix operations, on computers is an obvious observation. When I started my mathematical career (about 40 years ago), I was not aware that numerical linear algebra is also a (lively) mathematical research field. I have learned this meanwhile – to a large extent through my contacts with Volker Mehrmann and his research environment. The VM60 Festival seems to be a good opportunity to survey some of my experiences and encounters with numerical linear algebra, and in particular, to sketch the type of numerical linear algebra that is important for my own work.

## Behaviour of time-varying DAEs

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We consider the *behaviour* of time-varying DAEs

$$\ker_{\mathcal{S}} R\left(\frac{d}{dt}\right) := \left\{ w \in \mathcal{S} \mid w \text{ is a weak solution of } R\left(\frac{d}{dt}\right)w = 0 \right\}.$$

where

$$R(s) = R_0(\cdot) + R_1(\cdot)s + \dots + R_N(\cdot)s^N \in \mathcal{C}^{g \times q}[s]$$

and

$$\mathcal{S} \subset L_{\text{loc}}^1(I; \mathbb{R}^q) \text{ denotes a vector space, } I \subset \mathbb{R} \text{ an interval, } \mathcal{C} \text{ a function space.}$$

We will in particular study finite escape time, input-output systems, controllability, observability, and zero dynamics. The seminar is based on [1, 2, 3, 4]

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## On the geometric integration of self-adjoint linear DAEs

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Self-adjoint linear DAEs arise, e. g., in the necessary conditions for linear-quadratic optimal control problems with constraining linear DAEs or by linearization of DAEs from the modelling of multibody systems. Starting from local and global canonical forms for such structured problems, we show that under a suitable restricted class of transformations we are able to separate a hamiltonian system of differential equations. In this sense, we may say that a self-adjoint linear DAE exhibits a symplectic flow. Based on these observations, we will discuss a possibility for the geometric integration of self-adjoint linear DAEs. Techniques include structured index reduction, time-dependent transformations and automatic differentiation.

## Spectrum-based robust stability analysis of linear delay differential-algebraic equations

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In this talk we discuss recent results on the robust stability analysis of linear delay ordinary differential equations (DODEs) and linear delay differential-algebraic equations (DDAEs). We investigate whether the asymptotic/exponential stability of a given system is preserved when the system coefficients are subject to structured perturbations. In particular, we are interested in computing the distance (measured in an appropriate metric) between the nominal stable system and the closest perturbed systems that loses the stability. This quantity is called the distance to instability or the *stability radius* of the system. We focus on the spectrum-based stability criteria and the formulation of stability radii for linear delay systems. The stability and robust stability analysis for DAEs is quite different from that of ODEs since the system dynamics is constrained, see [1]. Not only the stability, but also some other DAE properties must be considered. If the time-delay is involved, then the existence and the behaviour of solutions become more complicated, see [2]. In the first part of the talk, we briefly overview important results on the stability radii for linear time-invariant DODEs and an extended result for linear time-varying DODEs. In the second part, we discuss some recent results on the spectrum-based stability and robust stability analysis for general linear time-invariant DDAEs [2]. We close the talk by mentioning some further related results and topics for future research.

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## Matrix polynomials in non-standard form

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Matrix polynomials  $P(\lambda)$  and their associated eigenproblems are fundamental for a variety of applications. Certainly the standard (and apparently most natural) way to express such a polynomial has been

$$P(\lambda) = \lambda^k A_k + \lambda^{k-1} A_{k-1} + \cdots + \lambda A_1 + A_0,$$

where  $A_i \in \mathbb{F}^{m \times n}$ . However, it is becoming increasingly important to be able to work directly and effectively with polynomials in the non-standard form

$$Q(\lambda) = \phi_k(\lambda)A_k + \phi_{k-1}(\lambda)A_{k-1} + \cdots + \phi_1(\lambda)A_1 + \phi_0(\lambda)A_0,$$

where  $\{\phi_i(\lambda)\}_{i=0}^k$  is some other basis for the space of all scalar polynomials of degree at most  $k$ . This talk will describe some new approaches to the systematic construction of families of linearizations for matrix polynomials like  $Q(\lambda)$ , with emphasis on the classical bases associated with the names Newton, Hermite, Bernstein, and Lagrange.

## Moving eigenvalues and eigenvectors - perturbation theory in practice

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In the context of iterative solvers moving the eigenvalue or the eigenpair may be of particular importance in several cases, e.g., deflation techniques, increasing the spectral gap or determining the set of linearly independent eigenvectors. It can also be used for reducing the imaginary parts of the eigenvalues without changing the matrix exponential; this can enhance the computation of  $\exp(A)$ . Exploiting the classical perturbation analysis for eigenvalue problems [2] we study the following problem.

Given a matrix  $A \in \mathbb{C}^{n \times n}$ , a simple eigenvalue  $\lambda$  and corresponding right and left eigenvector,  $x$  and  $y$ , such that

$$Ax = \lambda x \quad \text{and} \quad y^T A = \lambda y^T, \quad (1)$$

our goal is to obtain a perturbation  $\Delta A$  which allows moving the eigenvalue  $\lambda$  or/and the associated eigenvector  $x$  such that the other eigenvalues, as well as all right and left eigenvectors, will stay unaffected by the perturbation. Similar analysis is carried out for the generalized and quadratic eigenvalue problems [1].

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## Structured vs. unstructured spectral perturbation: a particular overview

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The design and analysis of structure-preserving algorithms to solve structured eigenproblems has led in the last decades to a steady interest in structured eigenvalue perturbation theory, i.e. in determining the behavior of eigenvalues and other spectral objects (e.g., invariant subspaces, sign characteristics,...) when a matrix or operator is subject to perturbations belonging to the same class of operators as the unperturbed one. It is well known that this behavior may be quite different from the behavior under arbitrary, nonstructured perturbations.

In this talk I attempt to give an overview of several results obtained in the last few years which show the peculiarities of structured vs. unstructured perturbations in a number of different contexts, e.g. spectral conditioning, low-rank perturbations or, if time allows, multiplicative ones. Several of these results are either due to Volker and co-authors, or have been motivated by his work on Control problems.

## Cyclic reduction and index reduction/shifting for a second-order probabilistic problem

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I wish to describe a problem that has many similarities with the differential-algebraic and boundary-value problems that appear in mechanics and control theory, although it has a different application background and different involved matrix structures.

*Markov-modulated Brownian motion* is a probabilistic process used in modelling a variety of real-life phenomena. The model consists in a real-valued stochastic process which evolves under a Brownian motion law whose parameters depend on the state of an underlying (environment) continuous-time Markov chain with  $n$  states. Its stationary distribution can be represented as a vector-valued function  $f : [0, \infty] \mapsto \mathbb{R}_{\geq 0}^n$  which satisfies the constant-valued differential-algebraic equation

$$\ddot{f}(x)V - \dot{f}(x)D + f(x)Q = 0, \quad (2)$$

where  $Q \in \mathbb{R}^{n \times n}$  is the generator matrix of a continuous-time Markov chain (a singular  $-M$ -matrix), while  $V \in \mathbb{R}_{\geq 0}^{n \times n}$  and  $D \in \mathbb{R}^{n \times n}$  (with mixed signs) are diagonal. Boundary conditions are at 0 and  $\infty$  (or 0 and  $M > 0$ , in some problems). A classical approach to solving (2) is identifying the invariant subspace associated to the stable eigenvalues of the matrix polynomial  $V\lambda^2 - D\lambda + Q$ . For instance, a normwise stable approach based on linearization + generalized Schur form exists in literature [1].

We describe an approach based on Cyclic Reduction, a famous matrix iteration, to solve this problem in a componentwise accurate way, relying on the sign properties of the involved matrices and using a special subtraction-free variant of Gaussian elimination, the *GTH method*. This work extends our previous research [2] on first-order problems (those with  $V = 0$ , known as *fluid queues*). Some novel features appear for second-order problems:

- Switching to a more general formulation with invariant pairs (instead of a matrix equation) is necessary to ensure the correct signs for subtraction-free methods.
- There is less freedom in the choice of the eigenvalue transformation map, an intermediate step that has some points in common with discretization methods for the solution of ODEs.
- In the cases in which  $V$  has zero diagonal entries, postmultiplication by a matrix pencil is used to modify the position of the infinite eigenvalues. This transformation can be interpreted as index reduction via differentiation of some equations; in addition to adjusting the eigenvalue positions, it plays an important role in getting the correct signs to ensure the applicability of the componentwise-accurate methods.

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## On ADI approximate balanced truncation

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Balanced truncation is one of the most popular model reduction methods for input-output-systems governed by ordinary differential equations. This technique relies on the determination of the observability and controllability Gramian matrices and provides an error bound in the  $H_\infty$  norm. For this method, a variety of efficient numerical methods have been developed in the past couple of years. In particular, the ADI iteration for determining the Gramians has become very popular since it allows to determine balanced realizations of large-scale systems.

Since ADI iteration provides approximative solutions, it is natural to wonder the effect of this approximation in the overall model reduction process. This is subject the talk, where we aim to present a backward error analysis: We first show that ADI approximate balanced truncation in theory consists of exact balanced truncation of a certain artificial system, which is obtained from the original system via an  $L^2$ -orthogonal projection of the impulse response. Numerical consequences will be presented.

## Decay pattern of matrices: application to matrix functions and matrix equations

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Sparsity and structural properties of matrices have a key role in the development of many efficient and stable algorithms. The entry decay pattern of matrices is also emerging as a helpful piece of information for analyzing and approximating complex problems. Indeed, a possibly exponential decay pattern away from the main diagonal can be observed and described for functions of matrices with particular structure. In this talk we review some available bounds, and also show new decay estimates for the entries of a wide class of matrix functions. We then tailor these bounds to matrices with Kronecker structure, as they arise in many application problems associated with partial differential equations. We also report on the use of these patterns in the numerical solution of linear matrix equations.

*Partly joint work with Michele Benzi, Emory University (USA)*

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## Model reduction of linear and nonlinear magneto-quasistatic equations

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The dynamic behaviour of electromagnetic devices can be described by Maxwell's equations coupled with circuit equations. In magneto-quasistatic problems, the contribution of the displacement currents is negligible compared to the conductive currents. A finite element discretization of Maxwell's equations in magnetic vector potential formulation combined with the circuit coupling equations yields a large system of differential-algebraic equations of special structure. For model order reduction of linear systems, we employ a balanced truncation approach, whereas nonlinear systems are reduced using a proper orthogonal decomposition method combined with a discrete empirical interpolation technique. We will exploit the special structure of the underlying problem to improve the performance of the model reduction algorithms.

## Asynchronous optimized Schwarz methods

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Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. Mathematical models of this computational paradigm were developed in the 1980s and 90s and convergence proofs given; see, e.g., the survey [1] and references therein.

Schwarz iterative methods were originally devised to show existence of solutions of elliptic problems on irregular domains and were revived as numerical methods in the 1980s; see, e.g., [4], and the many references therein. For these Schwarz methods, one may consider the solution of a general problem of the form

$$\begin{cases} \mathcal{L}(u) = f \text{ in } \Omega \\ \mathcal{C}(u) = g \text{ on } \partial\Omega, \end{cases} \quad (3)$$

where  $\mathcal{L}$  and  $\mathcal{C}$  a partial differential operators defined on the domain  $\Omega$  and its boundary, respectively. This domain is (artificially) split into two or more (possibly overlapping) subdomains, i.e., we have  $\Omega = \cup_{i=1,\dots,p} \Omega_i$ . In essence one is introducing new artificial boundary conditions on the interfaces between these subdomains. In the classical formulation, these artificial boundary conditions are of Dirichlet type. Given an initial approximation  $u(0)$ , the method progresses by solving for  $u(n+1)$  the equation (3) restricted to each subdomain  $\Omega_i$  using as boundary data on  $\delta\Omega_i \setminus \delta\Omega$  the values for  $u(n)$ . This procedure is inherently parallel, since the (approximate) solutions on each subdomain can be performed by a different processor.

Convergence of these iterations can be guaranteed under mild conditions, but it is in general rather slow, comparable to the Block Jacobi or Block Gauss-Seidel methods for linear algebraic systems. Much faster convergence can be achieved by using Robin and mixed boundary conditions on the interfaces. In this way one can optimize the Robin parameter(s) and obtain a very fast method. This technique has been termed optimized Schwarz methods; see, e.g., [2]. See also [3] for an algebraic version of this approach.

In this talk, an asynchronous version of the optimized Schwarz method is presented for the solution of differential equations of the form (3) on a parallel computational environment. In a one-way subdivision of the computational domain, with overlap, the method is shown to converge when the optimal artificial interface conditions are used. Convergence is also proved under very mild conditions on the size of the subdomains, when approximate (non-optimal) interface conditions are utilized. Numerical results are presented on large three-dimensional problems illustrating the efficiency of the proposed asynchronous parallel implementation of the method.

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## Modeling and numerical analysis of PDAEs describing flow networks

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The simulation of flow networks as electric circuits, water and gas supplying networks leads to partial differential algebraic equation systems (PDAEs). Depending on the flow medium and the model level, the systems contain partial differential equations of elliptic/parabolic/hyperbolic type and/or ordinary differential equations [1, 2, 3, 4]. They are coupled by linear constraints arising from the network topology.

We present some common structures of the resulting PDAE systems. Additionally, we demonstrate that the stability of numerical schemes is highly influenced by the constraints and present some suitable discretizations for certain prototype PDAEs. The presentation bases on joint work with C. Huck, L. Jansen, R. Lamour, R. März and M. Matthes.

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## Max-Balancing Hungarian Scalings

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A Hungarian scaling is a two-sided diagonal scaling of a matrix, which can be applied along with a permutation  $P$  to a linear system  $Ax = b$  with  $A \in \mathbb{C}^{n \times n}$  and  $b \in \mathbb{C}^n$  yielding

$$H = PD_1AD_2, \quad Hy = PD_1b, \quad x = D_2y,$$

where  $D_1, D_2 \in \mathbb{R}^{n \times n}$  are diagonal and nonsingular. The scaled and reordered matrix  $H = (h_{ij})$  is such that  $|h_{ij}| \leq 1$  and  $|h_{ii}| = 1$  for  $i, j = 1, \dots, n$ , and tends to be more diagonally dominant than the original matrix. Hungarian scaling improves the stability of LU factorization of sparse matrices and reduces the need for pivoting [2], [3]. It is an effective preprocessing step before applying preconditioned iterative methods [1].

We use max-plus algebra to characterize the set of all Hungarian scalings for a given matrix and show that max-balancing a Hungarian scaled matrix yields the most diagonally dominant Hungarian scaled matrix possible. We also propose an approximate max-balancing Hungarian scaling whose computation is embarrassingly parallel.

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## Fast and backward stable computation of the zeros of polynomials

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We present a fast and backward stable method for computing eigenvalues of upper Hessenberg unitary-plus-rank-one matrices, that is, matrices of the form  $A = \tilde{U} + \tilde{x}\tilde{y}^T$ , where  $\tilde{U}$  is unitary, and  $A$  is upper Hessenberg. This includes the class of Frobenius companion matrices, so this method can be used to find the zeros of a polynomial.

The unitary-plus-rank-one structure is preserved by any method that performs unitary similarity transformations, including Francis's implicitly-shifted  $QR$  algorithm. We present a new implementation of Francis's algorithm that acts on a data structure that stores the matrix in  $O(n)$  space and performs each iteration in  $O(n)$  time. The method is backward stable.

We store  $A$  in the form  $A = QR$ , where  $Q$  is unitary and  $R$  is upper triangular. In this sense our method is similar to one proposed by Chandrasekaran et. al. [1], but our method stores  $R$  differently. Since  $Q$  is a unitary upper-Hessenberg matrix, it can be stored as a product  $Q = Q_1 Q_2 \cdots Q_{n-1}$ , where each  $Q_j$  is a Givens-like unitary transformation that acts only on rows  $j$  and  $j + 1$ . We call these  $Q_j$  *core transformations*. Both our algorithm and that of Chandrasekaran et. al. use this representation of  $Q$ . For  $R$ , they use a quasiseparable generator representation. Our representation scheme factors  $R$  in the form

$$R = C_{n-1} \cdots C_1 (B_1 \cdots B_{n-1} + e_1 y^T),$$

where the  $C_j$  and  $B_j$  are unitary core transformations. This is possible because  $R$  is also unitary-plus-rank-one.

The Hessenberg matrix  $A$  takes the form

$$A = QR = Q_1 \cdots Q_{n-1} C_{n-1} \cdots C_1 (B_1 \cdots B_{n-1} + e_1 y^T),$$

and thus is represented by about  $3n$  core transformations plus the rank-one part. In fact there is some redundancy in the representation. The information about the rank-one part is also encoded in the core transformations, so it is not necessary to store the rank-one part explicitly. Performing a Francis iteration on a matrix stored in this form is entirely a matter of manipulating core transformations. We will show how to do this.

Our method is about as accurate as and much faster than the (slow) Francis algorithm applied to the companion matrix without exploiting the structure. It is faster than other fast and (allegedly) backward stable methods that have been proposed, and it has comparable or better accuracy.

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## Compressing the coefficient matrices of a singular matrix polynomial

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Following the idea in [1], we present a compressing procedure for singular matrix polynomials of the form

$$P(\lambda) = \sum_{j=0}^k \lambda^j A_j = \lambda^k A_k + \lambda^{k-1} A_{k-1} + \dots + \lambda A_1 + A_0.$$

where  $A_0, \dots, A_k \in \mathbb{C}^{m \times n}$ . The procedure applies a sequence of  $k-1$  unitary transformations simultaneously on the coefficient matrices to form a matrix polynomial

$$\tilde{P}(\lambda) = U^* P(\lambda) V =: \lambda^k \tilde{A}_k + \dots + \lambda \tilde{A}_1 + \tilde{A}_0,$$

where  $U, V$  are unitary and all the coefficient matrices are in block forms  $\tilde{A}_j = [A_{pq}^{(j)}]_{k \times k}$ . A so-called trimmed linearization  $\lambda \tilde{N} + \tilde{M}$  can be formed with the blocks from the transformed coefficient matrices, where

$$\tilde{N} = \begin{bmatrix} \tilde{A}_1 & A_{1:k,1:k-1}^{(2)} & A_{1:k,1:k-2}^{(3)} & \cdots & A_{1:k,1:2}^{(k-1)} & A_{1:k,1:1}^{(k)} \\ A_{1:k-1,1:k}^{(2)} & A_{1:k-1,1:k-1}^{(3)} & A_{1:k-1,1:k-2}^{(4)} & \cdots & A_{1:k-1,1:2}^{(k)} & \\ A_{1:k-2,1:k}^{(3)} & A_{1:k-2,1:k-1}^{(4)} & \ddots & \ddots & & \\ \vdots & \vdots & \ddots & \ddots & & \\ A_{1:2,1:k}^{(k-1)} & A_{1:2,1:k-1}^{(k)} & & & & \\ A_{1:1,1:k}^{(k)} & & & & & \end{bmatrix}$$

$$\tilde{M} = \begin{bmatrix} \tilde{A}_0 & & & & & \\ -A_{1:k-1,1:k-1}^{(2)} & -A_{1:k-1,1:k-2}^{(3)} & \cdots & -A_{1:k-1,1:2}^{(k-1)} & -A_{1:k-1,1:1}^{(k)} & \\ -A_{1:k-2,1:k-1}^{(3)} & -A_{1:k-2,1:k-2}^{(4)} & \cdots & -A_{1:k-2,1:2}^{(k)} & & \\ \vdots & \vdots & \ddots & & & \\ -A_{1:2,1:k-1}^{(k-1)} & -A_{1:2,1:k-2}^{(k)} & & & & \\ -A_{1:1,1:k-1}^{(k)} & & & & & \end{bmatrix},$$

and  $A_{1:p,1:q}^{(j)}$  is the submatrix of  $\tilde{A}_j$  on the leading  $p$  block rows and  $q$  block columns. This trimmed linearization preserves all the eigenstructure information of the original matrix polynomial, except the leading  $k-1$  Jordan chains of the eigenvalue infinity are deflated. In contrast, conventional linearizations may increase the length of singular chains [2, 3].

The compressing procedure can be simply applied to structured matrix polynomials for constructing structured trimmed linearizations.

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## **Abstracts of contributed talks**

## Fiedler-like pencils for rational eigenvalue problems

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Rational eigenvalue problems arise in many applications such as in acoustic emissions of high speed trains, calculations of quantum dots, free vibration of plates with elastically attached masses, vibrations of fluid-solid structures and in control theory. Therefore, computation of eigenvalues and eigenvectors of a rational matrix function is an important task which requires development of efficient numerical methods [1]. An obvious direct method to solve a rational eigenvalue problem (REP) is to transform the REP to a polynomial eigenvalue problem (PEP) by clearing out the denominators in the rational matrix function followed by linearization of the resulting PEP to obtain a generalized eigenvalue problem (GEP). Schematically,

$$\text{REP} \longrightarrow \text{PEP} \longrightarrow \text{GEP}.$$

A downside of this brute-force “*polynomialization*” of an REP is that the transformation from REP to PEP may introduce spurious eigenvalues, which may be difficult to detect and remove. Moreover, the transformation from REP to PEP followed by linearization may result in a GEP of very large dimension especially when the rational matrix function has a large number of poles. On the other hand, nonlinear eigensolvers such as Newton method and nonlinear Rayleigh-Ritz methods (e.g., nonlinear Arnoldi, rational Krylov, Jacobi-Davidson) may be suitable when a few eigenpairs are desired but their convergence analysis is a challenging task. A third alternative is to “linearize” an REP in an appropriate sense that avoids polynomialization of REP and provides a GEP of least dimension.

We propose “*linearization*” of a rational matrix function which extends linearization of a matrix polynomial to the case of a rational matrix function. We also introduce Fiedler-like matrix pencils for a rational matrix function and show that the Fiedler-like pencils are in fact linearizations of the rational matrix function. Further, we show that a Fiedler-like pencil  $\mathbb{L}_\sigma(\lambda)$  of a rational matrix function  $G(\lambda)$  associated with a permutation  $\sigma$  allows an easy recovery of eigenvectors of  $G(\lambda)$  from those of  $\mathbb{L}_\sigma(\lambda)$ .

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## Data-driven model reduction in the Loewner framework

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Interpolatory model reduction methods have matured quickly in the last decade and have been adopted by an ever-growing number of researchers. They have emerged as one of the leading choices for truly large scale problems. These methods have their roots in numerical analysis and linear algebra and are related to rational interpolation and Pade approximation. In the case of linear dynamical systems, the main idea behind these methods is to generate a reduced-model whose transfer function interpolates that of the original system at select interpolation points. Recently, major advances showed how to apply interpolation methods to nonlinear systems. The resulting approach turns out to global, in other words no small inputs are required.

In this talk we will give an overview of recent advances in *model reduction of linear and nonlinear dynamical systems* by means of *interpolatory methods* and in particular the *Loewner framework*. Several examples illustrating the theory will also be presented.

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# On the regularization of linear time-invariant descriptor systems

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For linear time-invariant descriptor systems we consider the question whether there exists a feedback which renders the closed-loop system regular. This property can be equivalently characterized by simple algebraic and geometric conditions in terms of the involved matrices and the augmented Wong sequences. We also consider the slightly more general problem of existence of a feedback such that an autonomous closed-loop system is obtained. The corresponding feedback matrices can be constructively obtained using a feedback canonical form [2].

For systems which are not regularizable by feedback, an additional behavioral equivalence transformation and a reorganization of input and state variables leads to a regular system, the index of which is at most one. This procedure is known [1], however we present a new approach which allows for a detailed characterization of the resulting regular system. We show that this system is fully determined by the augmented Wong sequences, which in particular allows for a simple calculation of the number of redundant equations, free state variables and constraint input variables independent of the transformation of the system.

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## Hierarchical vectors

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Adaptive mesh refinement is an important technique for handling equations that lead to solutions with localized features like shocks or singularities. Typically the construction of an adaptive mesh is based on an analysis of the underlying differential operators, e.g., residual error estimators typically rely on the coercivity of the corresponding bilinear form.

This talk presents an algebraic approach to mesh refinement: if the matrix describing the problem is rank-structured, e.g., if it is an  $\mathcal{H}^2$ -matrix, we can construct a hierarchical system of basis vectors that can be used to represent the solution. Choosing basis vectors from different levels of this hierarchy leads to an algebraic counterpart of a refined mesh.

If a vector can be represented by  $m$  hierarchical basis vectors, it is possible to compute the matrix-vector multiplication in  $\mathcal{O}(m)$  operations, but the result is represented in a matrix-dependent intermediate basis. We can use orthogonal projections to translate the result back to the original basis, and it is even possible to compute the corresponding projection error explicitly. This allows us to locally refine or coarsen the basis representation and leads to a purely algebraic refinement strategy.

## Dual pairs of Lyapunov inequalities

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In his inspiring paper [1], *Positive Operators and an Inertia Theorem*, Hans Schneider pointed out a close relationship between inertia theorems for Lyapunov equations and positive operators on the space of Hermitian matrices as well as the theory of  $M$ -matrices. Among other things, he showed that Lyapunov's matrix theorem can be extended to the case where a positive operator is added to the Lyapunov operator. This result turned out to be fundamental e.g. for the analysis of linear stochastic systems, see [2]. In typical applications it is interpreted as a criterion for a system to be asymptotically stable.

There is another famous result involving the Lyapunov operator (see e.g. [3, 4]), which plays an important role in model order reduction. For a system  $\dot{x} = Ax$  with associated Lyapunov operator  $L_A : X \mapsto AX + XA^*$ , it can be stated in the following equivalent forms, where

$$\Sigma = \text{diag}(\Sigma_1, \Sigma_2) > 0 \text{ with } \sigma(\Sigma_1) \cap \sigma(\Sigma_2) = \emptyset$$

is some block-diagonal matrix.

- (a) If  $L_A(\Sigma) \leq 0$  and  $L_A^*(\Sigma) \leq 0$ , then the projected subsystems corresponding to the blocks  $\Sigma_i$  are asymptotically stable.
- (b) If  $L_A(\Sigma) \leq 0$  and  $L_A(\Sigma^{-1}) \leq 0$ , then the projected subsystems corresponding to the blocks  $\Sigma_i$  are asymptotically stable.

It is immediate to formulate analogous generalized statements for the case, where a positive operator  $\Pi$  is added to the Lyapunov operator, that is for operators  $L_A + \Pi$  as considered in [1]. However, the generalizations of (a) and (b) are no longer equivalent and the proofs are less immediate than the statements. Some of the results appeared recently in [5].

In this talk we discuss applications to model order reduction and show the relation of our results to the theory of positive and cross-positive mappings (e.g. [6, 7, 8]). There are multiple connections to topics treated by Volker in his work.

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## Low rank perturbation of canonical forms

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Low rank modifications of a physical system that depends on many parameters arise when only a few parameters are modified, regardless of their magnitude (in the sense of norm). When the physical system is modeled by a system of linear differential (or differential-algebraic) equations of degree  $d$ :

$$A_d x^{(d)} + \cdots + A_1 x' + A_0 x = f, \quad A_0, A_1, \dots, A_n \in \mathbb{C}^{m \times n}, \quad (4)$$

then this kind of modifications result in low rank perturbations of the associated matrix polynomial  $A_0 + \lambda A_1 + \cdots + \lambda^d A_d$ .

It is of particular interest the case of linear differential-algebraic equations of degree 1:

$$Bx' + Ax = f, \quad A, B \in \mathbb{C}^{m \times n},$$

where the associated polynomial is a *pencil*,  $A + \lambda B$ .

The behavior of the solution of the equation (4) can be described using the canonical form of the matrix polynomial (the *Smith form* for general polynomials, and the *Kronecker canonical form* (KCF) for pencils, which, for regular pencils, is also known as the *Weierstrass canonical form* (WCF)). Hence, the study of how this canonical form changes after low rank perturbations is interesting, not only as a theoretical problem, but also in a practical setting.

In this talk, we will review known results that describe the change of the following canonical forms under low rank perturbations:

- The Jordan canonical form of a matrix [4, 5].
- The WCF of a regular pencil [3].
- The KCF of a singular pencil without full rank [1].
- The Smith form of a regular matrix polynomial [2].

We will also relate some of these results with recent work by Volker and collaborators that deal with structured matrices (like selfadjoint, symplectic, orthogonal, or unitary).

This talk is mainly based on joint work with F. M. Dopico and J. Moro.

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## Orbit closure hierarchies of skew-symmetric matrix pencils

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We study how small perturbations of a skew-symmetric matrix pencil may change its canonical form under congruence. This problem is also known as the stratification problem of skew-symmetric matrix pencil orbits and bundles. In other words, we investigate when the closure of the congruence orbit (or bundle) of a skew-symmetric matrix pencil contains the congruence orbit (or bundle) of another skew-symmetric matrix pencil. The developed theory relies on our main theorem stating that a skew-symmetric matrix pencil  $A - \lambda B$  can be approximated by pencils strictly equivalent to a skew-symmetric matrix pencil  $C - \lambda D$  if and only if  $A - \lambda B$  can be approximated by pencils congruent to  $C - \lambda D$ . The stratification theory is also illustrated by using StratiGraph.

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## Volker Mehrmann and modern factorizations of symplectic matrices

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Theory and structured algorithms concerning symplectic matrices are among main Volker's research interests along his whole career. Many of his papers and books contain results and algorithms related to this fundamental group of matrices and its applications to systems and control theory, as well as to classical mechanics and Hamiltonian dynamical systems, see for instance [6, 7, 5] and many other references therein. A technique that Volker has very often used in his theoretical and numerical work on symplectic matrices is to factor a general symplectic matrix into finite products of simpler symplectic matrices that reveal important properties of the original symplectic matrix. This strategy has deeply influenced the research in this area (see [3] and the references therein) and, in particular, a simple but fundamental symplectic factorization lemma proved by Volker in late 1980's was the starting point of my papers [1, 2]. In addition, this symplectic factorization strategy is also the main thread of the unpublished survey-biased manuscript [4], coauthored by Steven Mackey, Nil Mackey, and Volker, which is also closely connected with [1, 2]. This talk has two main purposes: first, to summarize some key factorizations of symplectic matrices developed by several authors since the 1980's, with special attention to those included in [1, 2, 4]; and, second, to encourage Steven Mackey, Nil Mackey, and Volker to finish and submit the manuscript [4].

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## Discrete input/output maps and a generalization of the proper orthogonal decomposition method

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Current control design techniques require system models of moderate size to be applicable. The generation of such models is challenging for complex systems which are typically described by partial differential equations (PDEs), and model-order reduction or low-order-modeling techniques have been developed for this purpose. Many of them heavily rely on the state space models and their discretizations. However, in control applications, a sufficient accuracy of the models with respect to their input/output (I/O) behavior is typically more relevant than the accurate representation of the system states.

In this talk, we present a discretization framework which has been developed recently [1] and which heavily focuses on the I/O map of the original PDE system. In particular, the proposed direct discretization of the I/O map of a linear time-invariant system comes with error bounds measuring the relevant I/O error. We show how the discretized I/O map can be realized as a matrix. By tensor techniques the I/O matrix can be further reduced to a very low-dimensional map which is shown to be beneficial in a control application.

For special choices of input and output spaces, the proposed reduction coincides with the well-known Proper Orthogonal Decomposition (POD) method. Turning this argument around, we find that the method of discretizing I/O maps can be employed for a generalization of the common POD method. We present numerical examples [2] that demonstrate the benefits of generalized POD.

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## Estimating the traces of powers of certain nonnegative matrices related to orthogonal polynomials

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In [1], inequalities satisfied by the traces of powers of a special nonnegative matrix arising in the study of the nonnegative inverse eigenvalue problem are found and results on the coefficients of certain related power series derived. Here, we answer similar questions for certain patterned matrices. Let  $P_n$  be the  $n \times n$  permutation matrix corresponding to the cycle  $(1\ 2\ \dots\ n)$  and let  $C'_n$  be the circulant  $P_n + P_n^{-1}$ . Using power series, we study the traces of the powers of  $C'_n$  and several related matrices associated with orthogonal polynomials. The expansion in powers of  $t$  of

$$\left( \frac{2}{1 + \sqrt{1 - 4t^2}} \right)^c$$

plays a fundamental role.

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## On the backward stability of computing polynomial roots via colleague matrices

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Computing the roots of scalar and matrix polynomials expressed in the Chebyshev basis  $\{T_k(x)\}$  is a fundamental problem that arises in many applications. For instance, a standard way to compute the real roots of a smooth function  $f(x)$  on an interval is to approximate  $f(x)$  by a polynomial  $p(x)$  via Chebyshev interpolation. A common way of computing the roots of a polynomial expressed in the Chebyshev basis is to compute the eigenvalues of its colleague matrix. In this work, we analyze the backward stability of the polynomial root-finding problem solved with colleague matrices. In other words, given the polynomial  $P(x) = T_n(x) + \sum_{k=0}^{n-1} A_k T_k(x)$ , with  $A_k \in \mathbb{R}^{p \times p}$ , expressed in the Chebyshev basis, the question is to determine whether the whole set of computed eigenvalues of the colleague matrix, obtained with a backward stable algorithm like the QR-algorithm for the standard eigenvalue problem, are the set of roots of a nearby polynomial or not. This question was answered by A. Edelman and H. Murakami in [1] when the polynomial is expressed in the monomial basis. In this work, we derive a first order backward error analysis of the polynomial root-finding algorithm using colleague matrices following a different approach to the one followed by A. Edelman and H. Murakami. Our backward error analysis expands on a very recent work by Y. Nakatsukasa and V. Noferini [2] in that we show that this algorithm is backward normwise stable if the coefficients of the polynomial  $P(x)$  have moderate norms. We also present numerical experiments that support these theoretical results.

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## Eigenvalues of rank one perturbations of matrices with structure in an indefinite inner product space

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The effect of a generic rank one perturbation of a general matrix on the Jordan structure has been studied in several papers. The generic result is that for each eigenvalue the largest Jordan block is split into simple eigenvalues, while the other Jordan blocks corresponding to that eigenvalue remain (the Jordan basis changes of course). See [1, 7, 8].

In the talk we consider the effect of a structured but otherwise generic rank one perturbation on the eigenvalues and Jordan structure of a matrix that has a symmetry property in an indefinite inner product space. An overview will be given of some of the main results obtained in this area. Compared to the unstructured case there are some surprises, and we shall focus on what is different from the unstructured case. These differences are typically connected to paired Jordan blocks corresponding to specific eigenvalues in the canonical forms for such classes of matrices.

The talk is based on joint work with Volker Mehrmann, Christian Mehl and Leiba Rodman, [2, 3, 4, 5, 6].

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## A generalized Krylov subspace method for $\ell_p$ - $\ell_q$ minimization

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We present a new efficient approach for the solution of the  $\ell_p$ - $\ell_q$  minimization problem based on the application of successive orthogonal projections onto generalized Krylov subspaces of increasing dimension. The subspaces are generated according to the iteratively reweighted least-squares strategy for the approximation of  $\ell_p/\ell_q$ -norms by weighted  $\ell_2$ -norms. Computed image restoration examples illustrate that it suffices to carry out only a few iterations to achieve high-quality restorations. The combination of a low iteration count and a modest storage requirement makes the proposed method attractive.

## On the Vorobyev method of moments

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In 1958 Yu. V. Vorobyev published (in Russian) a book called (in English translation that appeared in 1965) *Method of Moments in Applied Mathematics*. As mentioned in the annotation, “This book presents the theory behind the moment method for finding the eigenvalues of a linear operator approximately and for solving linear problems.” This book remained within the mathematical community almost unnoticed. Its importance has been pointed out by Claude Brezinski in 1996 in relation to the Lanczos method. Influenced by Gene Golub (through his interest in moments), by Volker Mehrmann (through the discussions on model reduction in control and in PDEs) and others, the author of this contribution would like to recall some ideas of Vorobyev in relation to some recent context.

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## Rank structures in multidimensional matrices

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I would like to discuss the differences between the new representation formats for multidimensional matrices, in particular TT and HT formats, and the lines of further development of the cross approximations techniques. Besides that, we consider some new applications to numerical solution of the Smoluchowski-like equations and the parameter identification problem for some models of biological systems.

## Generalization of Lagrange linearization for polynomial eigenvalue problems

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Let  $P(z)$  be a polynomial  $n \times n$  matrix. Consider the following *polynomial eigenvalue problem*: look for nonzero vectors  $v$  (right eigenvectors) and corresponding eigenvalues  $\lambda$  such that the following equation is satisfied

$$P(\lambda)v = 0.$$

A *linearization* of this eigenvalue problem is a square polynomial matrix  $L(z)$  of degree 1 such that

$$U(z)L(z)V(z) = \begin{bmatrix} P(z) & 0 \\ 0 & I \end{bmatrix}$$

with  $U(z)$  and  $V(z)$  unimodular polynomial matrices. Several linearizations have appeared in the literature based on the representation of the polynomial matrix  $P(z)$  in different bases, e.g., degree graded bases such as the monomial basis, the Chebyshev basis, ..., or interpolation bases, such as the Lagrange polynomials.

If the polynomial matrix  $P(z)$  has degree  $N$ , this matrix is uniquely determined by its values  $P_i$  in  $N + 1$  points  $\sigma_i$ ,  $i = 0, 1, 2, \dots, N$ , i.e.,  $P_i = P(\sigma_i)$ . A Lagrange-type linearization [1] based on this representation is

$$L(z) = \begin{bmatrix} 0 & P_0 & \cdots & P_N \\ -\beta_0 I_n & (z - \sigma_0)I_n & & \\ \vdots & & \ddots & \\ -\beta_N I_n & & & (z - \sigma_N)I_n \end{bmatrix}$$

where the  $\beta_i$  are the so-called barycentric weights. In this talk, we will generalize such linearizations allowing to use value-information of the polynomial matrix in  $(N + 1)n$  different points which could have a beneficial effect on the conditioning of the corresponding eigenvalue problem.

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## Structured backward stability of linearizations of polynomial matrices

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In this talk we discuss the structured backward stability of linearizations of a given polynomial matrix  $P(\lambda)$  that can be given either in: (i) the classical monomial basis, (ii) the Chebyshev basis, or, (iii) the barycentric Lagrange basis with given interpolation points. We show that for these different classes of linearizations, running the QZ algorithm on the linearized pencil yields a relative backward error on the pencil that can be mapped back to a relative backward error of essentially the same size on the coefficients of the original representation. In that sense we can say that the computed roots correspond exactly to the roots of a nearby polynomial matrix where “nearness” has to be interpreted in the coefficient space of the representation being used. The proofs of these results rely, to a certain extent, on the concept of dual minimal bases as developed in [1], [2].

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## **Abstracts of posters**

## Regularization of operator DAEs

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A general framework for the regularization of constrained PDEs, also called operator differential-algebraic equations (DAEs), is presented. For this, we consider semi-explicit systems of first order which includes the Navier-Stokes equations [1].

The proposed reformulation is consistent in the sense that the solution of the PDE remains untouched. However, one can observe improved numerical properties in terms of the sensitivity to perturbations and the fact that a spatial discretization leads to a DAE of lower index, i.e., of differentiation index 1 instead of differentiation index 2.

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## Nested Krylov methods for shifted linear systems

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Several applications require the solution of a sequence of shifted linear systems of the form

$$(A - \omega_k I)\mathbf{x}_k = \mathbf{b}, \quad (5)$$

where  $A \in \mathbb{C}^{N \times N}$ ,  $\mathbf{b} \in \mathbb{C}^N$ , and  $\{\omega_k\}_{k=1}^n \in \mathbb{C}$  is a sequence of  $n$  distinct shifts. For example, shifted linear systems arise in model order reduction as well as in the geophysical exploration of both acoustic and elastic waves.

In our application, we focus on wave propagation through elastic media in a frequency-domain formulation. This formulation has specific advantages when modeling visco-elastic effects. In order to improve the imaging of the earth crust, so-called *full waveform inversion* is computed which is an optimization problem at multiple wave frequencies. Therefore, the grid size must be small enough to describe the wave, which for high frequencies results in very large shifted linear systems of the form (5).

In principle, a sequence of shifted systems (5) can be solved almost at the cost of a single solve using so-called shifted Krylov methods. These methods exploit the property that Krylov subspaces are invariant under arbitrary diagonal shifts  $\omega$  to the matrix  $A$ , i.e.,

$$\mathcal{K}_m(A, \mathbf{b}) = \mathcal{K}_m(A - \omega I, \mathbf{b}), \quad \forall m \in \mathbb{N}, \forall \omega \in \mathbb{C}. \quad (6)$$

However, in practical applications, the preconditioning of (5) is required which in general destroys the shift-invariance property (6). In [1], a polynomial preconditioner that preserves the shift-invariance is suggested. The presented work [2] is a new approach to the iterative solution of (5). We use *nested* Krylov methods that use an inner multi-shift Krylov method as a preconditioner for a flexible outer Krylov iteration. In order to deal with the shift-invariance, our algorithm assumes the inner Krylov method to produce collinear residuals for the shifted systems. In my presentation, I will concentrate on two possible combinations of Krylov methods for the nested framework, namely FOM-FGMRES and IDR-FQMRIDR. Since residuals in multi-shift IDR are not collinear by default, the development of a collinear IDR variant which is suitable as an inner method in the new framework is a second main contribution of our work.

An extension of [2] to shifted systems with multiple right-hand sides  $B \equiv [\mathbf{b}_1, \dots, \mathbf{b}_\ell]$ ,  $\ell \ll N$ , using block Krylov methods is subject to our current research.

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## Perturbation theory of the Moore-Penrose inverse and the least squares problem

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The Moore-Penrose pseudo-inverse of an arbitrary matrix has many applications in numerical computation, statistics, control systems, curve fitting, and differential algebraic equations [5]. It is particularly useful in dealing with linear least squares problems  $\min_x \|b - Ax\|_2$ , see [1, 4], and very recently the error analysis of some highly accurate numerical algorithms presented in [3] for structured least square problems has been based on new perturbation expressions and bounds for the variation of the Moore-Penrose inverse. In this talk we first discuss some existing results for the additive and multiplicative perturbation of the Moore-Penrose pseudo-inverse [2] and, second, we extend the perturbation results that we introduced in [3] to obtain new perturbation bounds using unitarily invariant norms and Q-norms that improve significantly previous bounds available in the literature. We will comment on future research on accurate solutions of non-negative constrained least squares problems.

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## Ultrametric matrices and the geometric inverse $M$ -matrix problem

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Keywords: 0/1-matrix, 0/1-simplex, Stieltjes matrix, ultrametric matrices,  $M$ -matrix,  $n$ -simplex.

We describe the symmetric inverse  $M$ -matrix problem from a geometric viewpoint. The central question in the geometric context is, which properties the lower dimensional facets of an  $n$ -simplex  $S$  guarantee that  $S$  itself has no obtuse dihedral angles. The simplest but strongest of such properties is regularity of the triangular facets. Slightly weaker is to demand that all triangular facets are strongly isosceles. Even more general but also more involved is to demand ultrametricity of all threedimensional tetrahedral facets of  $S$ .

As part of our exposition we show that either none, or all so-called vertex Gramians associated with an  $n$ -simplex  $S$  are ultrametric. As a result, the inverse of an ultrametric matrix is weakly diagonally dominant if and only if this inverse is a Stieltjes matrix. Thus, only one of them needs to be proved in order to obtain both.

## A posteriori error estimates for a class of differential algebraic equations in singular perturbation context

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The present work considers two *a posteriori* error estimates generation for a class of differential algebraic equation(DAE)s on singular perturbation problem(SPP)s context. It is well known that the first challenge to solve a differential algebraic equation is to find a suitable consistent initial condition [4]. However, one can avoid this by making the problem more stiff, which can be done by introducing a small parameter (known as perturbation parameter) with the derivatives coefficient. In this case, the solution can have boundary layers. Therefore, the existing numerical analysis with fixed number of mesh points does not converge on uniform step size. This is because the number of mesh points need to be proportional to the inverse power of perturbation parameter for the convergence of the solution. The aim of this contribution is to provide *a posteriori* error estimates (both linear and of higher order accuracy) on fixed number points, which works for DAEs as well as for SPPs.

As the model problem, we consider the following two problems on  $t \in \Omega = (t_0, T]$ :

$$\left\{ \begin{array}{l} \mathbf{x}'_1(t) = \mathbf{f}_1(t, \mathbf{x}_1(t), \mathbf{x}_2(t)), \\ \mathbf{f}_2(t, \mathbf{x}_1(t), \mathbf{x}_2(t)) = 0, \\ \text{with consistent Initial Conditions,} \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \mathbf{y}'_1(t) = \mathbf{f}_1(t, \mathbf{y}_1(t), \mathbf{y}_2(t)), \\ \mathbf{eps} \mathbf{y}'_2 = \mathbf{f}_2(t, \mathbf{y}_1(t), \mathbf{y}_2(t)), \\ \text{with any Initial Conditions.} \end{array} \right. \quad (7)$$

Here,  $\mathbf{x}_1(t), \mathbf{y}_1(t) \in \mathbb{R}^{n_1}$  and  $\mathbf{x}_2(t), \mathbf{y}_2(t) \in \mathbb{R}^{n_2}$  for some positive integers  $n_1$  and  $n_2$  with  $n_1 + n_2 = l$  and  $(\mathbf{eps})_{n_2 \times n_2} = \text{diag}(\epsilon, \dots, \epsilon)$  with  $0 < \epsilon \ll 1$ . As the limiting process of  $\mathbf{y}(t)$ , we will get an approximate of  $\mathbf{x}(t)$ . The problem on the right hand side will be solved in singular perturbation context as the solution derivatives are unbounded, in general.

We have proposed two *a posteriori* error estimates (1st and 2nd order) for the above two problems. The key idea of the *a posteriori* analysis is to use the stability of the continuous solution via an M matrix condition. The right hand problem is solved in more general sense where the perturbation parameters are of different magnitude  $(\mathbf{eps})_{l \times l} = \text{diag}(\epsilon_1, \dots, \epsilon_l)$  (see for e.g., [2]). The error analysis is based on the adaptive moving mesh algorithm [3] via the mesh equidistribution principle [1] which starts with an error monitor function and distributes the error in a way so that each subinterval has same error measurement. The main challenge is to provide an *a posteriori* monitor function, whose equidistribution converges to a layer adapted mesh. It is shown that the adaptive mesh will lead to a uniform mesh if the solution derivatives are all bounded independent of the perturbation parameters. Therefore, this analysis will easily work for the non singularly perturbed ordinary differential equations. The present technique does not need the *a priori* information about the solution (like the location and width of boundary layers). Theoretically, we have shown that the numerical solution converges uniformly to the exact solution.

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## SOR-like methods for solving the Sylvester equation

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We present new iterative methods for solving large-scale Sylvester equation ( $AX - XB = C$ ). The proposed algorithms belong to the class of SOR-like methods, based on the SOR (Successive Over-Relaxation) method for solving linear systems (the first of the methods was proposed by Z. Woźnicki). All three are stationary iterative methods for solving  $AX - XB = C$ . We discuss convergence characteristics of the methods and present sufficient conditions under which proposed method ISOR-like is convergent.

We also present an idea of changing the given matrices  $A$  and  $B$  such that  $C$  and solution  $X$  remain the same, but the convergence of any SOR-like method is improved.

Some numerical experiments are given to illustrate the theoretical results and some properties of the methods.

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## A structure exploiting infinite Arnoldi exponential integrator for linear inhomogeneous ODEs

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Structure preserving and structure exploiting iterative methods have recently received considerable interest in the numerical linear algebra community; see e.g., [1]. Exponential integrators that use Krylov approximations of matrix functions have turned out to be efficient for the time-integration of certain ordinary differential equations (ODEs). In this result we will propose a new structure exploiting iterative method based on an Arnoldi method and exponential integrators to solve certain types of ODEs. We consider linear stiff inhomogeneous ODEs,  $y'(t) = Ay(t) + g(t)$ , where the function  $g(t)$  is assumed to satisfy certain regularity conditions. We derive an algorithm for this problem which is equivalent to Arnoldi's method. The construction is based on expressing the function  $g(t)$  as a linear combination of given basis functions  $[\phi_i]_{i=0}^{\infty}$  with particular properties. The properties are such that the inhomogeneous ODE can be restated as an infinite-dimensional linear homogeneous ODE. Moreover, the linear homogeneous infinite-dimensional ODE has properties that allow us to directly extend a Krylov method for finite-dimensional ODEs. Although the construction is based on an infinite-dimensional operator, the algorithm can be carried out with operations involving matrices and vectors of finite size. This type of construction resembles in many ways the infinite Arnoldi method, for nonlinear eigenvalue problem [2]. We prove convergence of the algorithm under certain natural conditions, and illustrate properties of the algorithm with examples stemming from the discretization of partial differential equations.

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## H-FAINV: Hierarchically factored approximate inverse preconditioners

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Given a sparse matrix, its LU-factors, inverse and inverse factors typically suffer from substantial fill-in, leading to non-optimal complexities in their computation as well as their storage. In the past, several computationally efficient methods have been developed to compute approximations to these otherwise rather dense matrices. Many of these approaches are based on approximations through sparse matrices, leading to well-known ILU, SPAI (sparse approximate inverse) or FSAI (factored sparse approximate inverse) techniques and their variants. A different approximation approach is based on blockwise low rank approximations and realized, for example, through hierarchical ( $\mathcal{H}$ -) matrices. While  $\mathcal{H}$ -inverses and  $\mathcal{H}$ -LU factors have been discussed in the literature, this paper will consider the construction of an approximation of the factored inverse through  $\mathcal{H}$ -matrices ( $\mathcal{H}$ -FAINV). We will describe a blockwise approach that permits to replace (exact) matrix arithmetic through approximate efficient  $\mathcal{H}$ -arithmetic. We conclude with numerical results in which we use approximate factored inverses as preconditioners in the iterative solution of the discretized convection-diffusion problem.

## On the comparison of sufficient conditions for the real and symmetric nonnegative inverse eigenvalue problems

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The real nonnegative inverse eigenvalue problem (RNIEP) is the problem of characterizing all possible real spectra of entrywise nonnegative matrices. This problem remains unsolved. Since the first result in this area announced by Suleimanova in 1949 and proved by Perfect in 1953, a number of realizability criteria or sufficient conditions for the existence of a nonnegative matrix with a given real spectrum have been obtained, from different points of view. In [2] the authors construct a map of sufficient conditions for the RNIEP, in which they show inclusion or independency relations between these conditions.

If in the RNIEP we require that the nonnegative matrix be symmetric, we have the symmetric nonnegative inverse eigenvalue problem (SNIEP). The first known sufficient condition for the SNIEP is due to Perfect and Mirsky in 1965 for doubly stochastic matrices, and Fiedler gave in 1974 the first symmetric realizability criteria for nonnegative matrices. It is well known that these two problems are equivalent for spectra of size  $n \leq 4$  and a complete solution of both is known only for  $n \leq 4$ . For  $n \geq 5$  they are different and both problems remain open.

Given a real spectrum  $\sigma$  verifying a sufficient condition  $X$ , we introduce the  $X$ -margin of realizability of  $\sigma$  to measure how much we can decrease the spectral radius of  $\sigma$  preserving the sufficient condition  $X$ . We analyze several sufficient conditions from the point of view of their margin of realizability [1]. Since 2007 new sufficient conditions for the RNIEP have appeared. We discuss new relations of inclusion or independency between these new sufficient conditions and the previous ones studied in [3]. We also construct a map of sufficient conditions for the SNIEP [4]. Finally, we describe and discuss some open problems of interest in this context.

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## The waveguide eigenvalue problem and the tensor infinite Arnoldi method

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We consider the following PDE–eigenvalue problem, which arises in the study of waves traveling in a periodic medium [1]: determine a non-trivial function  $u(x, z)$  and a complex number  $\gamma$  such that

$$\Delta u(x, z) + 2\gamma u_z(x, z) + (\gamma^2 + \kappa(x, z)^2)u(x, z) = 0, \quad (x, z) \in \mathbb{R}^2, \quad (8a)$$

$$u(x, z) = u(x, z + 1) \text{ for all } (x, z) \in \mathbb{R}^2, \quad (8b)$$

$$u(x, \cdot) \rightarrow 0 \text{ when } |x| \rightarrow \infty. \quad (8c)$$

The function  $\kappa(x, z)$  is piecewise constant and is assumed to satisfy:  $\kappa(x, z) = \kappa_-$  when  $x \leq x_-$ ,  $\kappa(x, z) = \kappa_+$  when  $x \geq x_+$  and  $\kappa(x, z) = \kappa(x, z + 1)$ . This problem can be rephrased as an equivalent problem on a finite domain by means of a Dirichlet–to–Neumann map. A particular type of finite-element discretization of the finite-domain problem leads to the following nonlinear eigenvalue problem, which consists of finding pairs  $(\gamma, v) \in \mathbb{C} \times (\mathbb{C}^n \setminus \{0\})$  such that

$$\begin{pmatrix} Q(\gamma) & C_1(\gamma) \\ C_2^T & R\Lambda(\gamma)R^{-1} \end{pmatrix} v = 0. \quad (9)$$

The matrices  $Q(\gamma)$  and  $C_1(\gamma)$  are polynomials of second degree in  $\gamma$ . The matrix  $\Lambda(\gamma)$  is diagonal and involves square roots of polynomials in  $\gamma$ . The problem (9) is a large-scale nonlinear eigenvalue problem of the type extensively studied in recent literature [2]. The algorithm we propose is based on the infinite Arnoldi method [3], which can be interpreted as the standard Arnoldi method applied to a linear and infinite dimensional eigenvalue problem. In the new algorithm, we suggest to represent the basis of the Krylov subspace as a factorization involving a tensor. This factorization allows us to reduce the memory requirements and the computation time. By construction, this new algorithm, which we call the tensor infinite Arnoldi method, is mathematically equivalent to the infinite Arnoldi method. The infinite Arnoldi method requires efficient procedures to compute the derivatives of the functions that define the nonlinear eigenvalue problem. For this problem such derivatives can be computed with a closed and efficient formula. Moreover we exploit sparsity and low-rank structure of the nonlinear eigenvalue problem. The matrix-vector product corresponding to  $R$  and  $R^{-1}$  can be computed with the Fast Fourier Transform (FFT).

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## Drugs, herbicides and numerical simulations

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Glyphosate is one of the herbicides used by the Colombian government to spray coca fields. Sprays took place for a number of years and were more frequent between 2000 and 2006. The spray drifts at the Ecuador-Colombia border became an issue for people living close to the border. We propose a mathematical model for the Glyphosate aerial spray drift at the Ecuador-Colombia border. The model takes into account the particular guidelines that aircrafts follow to perform the sprays. Numerical simulations in 2D and 3D are performed at sensitive zones along the Ecuador-Colombia border. The lack of reliable information constrains the accuracy of the model. However, the results presented in this work can be used as a starting point for more accurate models of the phenomena.

## Modeling of the crosstalk phenomenon for electro-magnetic systems by bilateral coupling of PDEs and DAEs

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Modeling the electro-magnetic disturbances between electrical elements which are sitting in an electro-magnetic system, the so-called crosstalk phenomenon, may lead to differential-algebraic equations (DAEs). In this poster, we propose a new modeling approach to describe the crosstalk phenomenon for electro-magnetic systems and, in particular, in electrical circuits by bilateral coupling of two sets of equations. The first set is a set of DAEs, the so-called circuit equations, i.e., the Kirchhoff current law as well as constitutive laws of the inductors and the voltage sources and presented in our model in the framework of modified nodal analysis, e.g. [1]. The second set is the set of partial differential equations (PDEs), the so-called non-stationary Maxwell equations, e.g. [2], modeling the induction of the electro-magnetic disturbances. Considering these two sets of equations as input-output subsystems, the bilateral coupling approach connects the output of one subsystem to the input of the other subsystem and conversely, via coupling and re-coupling relations. These relations, which are in principle physical constitutive laws, are introduced in our model by suitable operator structures [3]. The coupling of these two sets of equations leads to a set of partial differential-algebraic equations as the model equations for the crosstalk phenomenon in electrical circuits.

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## The sign characteristic of Hermitian matrix functions

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In the landmark paper [1], I. Gohberg, P. Lancaster and L. Rodman introduced and developed the theory of sign characteristic of Hermitian matrix polynomials with nonsingular leading coefficients. In this talk, we extend the theory to any Hermitian matrix polynomial. We show that a signature constraint theorem still holds. We also analyze in detail the consequences on the perturbation theory of regular selfadjoint matrix functions, and we give some examples of the applications of the new results.

This talk is based on joint work with V. Mehrmann, F. Tisseur, and H. Xu.

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## On single projection Kaczmarz extended type algorithms

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The Kaczmarz Extended (KE) algorithm has been proposed by the author in [2, 3] as an extension of the Kaczmarz-Tanabe algorithm from [4], to inconsistent linear least squares problems. It uses in each iteration orthogonal projections onto the hyperplanes determined by all the rows and all the columns of the system matrix. Recently, in the paper [5], the authors proposed a single projection KE type algorithm, which in each iteration uses orthogonal projections onto the hyperplanes determined by only one row and one column. If the projection row and column indices  $i$  and  $j$  are selected *at random with probability proportional with a certain quotient of the norm of the  $i$ -th row, and  $j$ -th column*, respectively, they prove that the sequence of approximations so generated converges in Expectation to a least square solution of the problem.

In this paper we propose two single projection KE type algorithms, in which the projection indices are selected in an *almost cyclic*, and *remote control* maner, respectively (see e.g. [1]). We prove that the sequence of approximations generated in each case converges in norm to a least square solution of the problem.

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## Complex Jacobi matrices and Gauss quadrature for quasi-definite linear functionals

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The Gauss quadrature rule is a method for the approximation of positive-definite linear functionals. The link between Gauss quadrature, orthogonal polynomials theory and (real) Jacobi matrices is well-known. We show a way to generalize the concept of Gauss quadrature for the approximation of quasi-definite linear functionals. To achieve this result we need to introduce the concept of complex Jacobi matrix as define in [1] by Beckermann. The generalization of Gauss quadrature still maintains a relationship with orthogonal polynomials theory and with the complex Jacobi matrices. Furthermore, this result is linked with the approximation through Krylov methods of bilinear forms such as  $\mathbf{u}^* f(A) \mathbf{v}$ , where  $A$  is a matrix,  $\mathbf{u}, \mathbf{v}$  two vectors and  $f$  a matrix function. In future we are going to work on this and analyze some possible applications, for example the approximation of *centrality indices* in the *complex networks theory*.

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## Dimensionality reduction for studying physical phenomena: case study with a brake squeal problem

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We explore dimensionality reduction in the context of model based and model free approaches. In the model based approach there is a governing equation or a set of rules relating quantities of interest, whereas in a model free setting there are no rules or equations, only data is available. As an example consider the problem of squealing noise in a brake. The model based approach relates quantities of interest like mass distribution within the brake, damping, stiffness and other properties of a brake material, speed of rotation etc with a dynamical equation, the steady state behaviour can be obtained by converting it to an eigenvalue problem and finding eigenvalues and eigenvectors (which are related to squeal frequency and mode shapes of a disc brake). The model reduction problem could be posed as projecting the eigenvalue problem to a lower dimensional space, while preserving important eigenvalues and eigenvectors. In contrast, the model free approach starts with data, i.e., a set of parameter values which correspond to squeal and the values which correspond to no-squeal. If the number of parameters responsible for squeal is very large, then dimensionality reduction is concerned with reducing the number of these parameters or ranking these parameters in order of importance. We illustrate pros and cons of model based and model free dimensionality reduction with some numerical examples.

## High-order adaptive sampling of parametric eigenvalue problems – application to photonic crystal bandstructure calculation

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We consider parameter dependent matrix eigenvalue problems for partial differential equations (PDEs) discretised by the finite element method (FEM). For an analytic dependency of eigenfunctions and eigenvalues on the parameter, let it  $k$ , we propose an adaptive strategy for the parameter sampling based on first and higher derivatives of the eigenvalues [1]. We obtain the derivatives for a single parameter value post-processing an eigenvalue and the corresponding eigenvector. For each second and higher derivative a sparse linear system of equations has to be solved. Then, with Taylor's theorem an high-order approximation of the eigenvalue in dependence of the parameter is given. Estimating the residual of the Taylor expansion we decide for a local step size  $h(k)$  and solve the eigenvalue problem at  $k + h(k)$ . In this way for each band  $\omega_n(k)$  an own adaptive sampling strategy is chosen. Evaluating the derivatives  $\omega'_j(k_c)$ ,  $\omega'_{j+1}(k_c)$  we can verify if on  $k = k_c$  two bands cross indeed each other or if they almost touch each other (mini-stop band). In the latter the algorithm re-started at  $k = k_c$  refines adaptively around the mini-stop band.

The algorithm is applied for the calculation of bandstructure of photonic crystals and photonic crystal wave-guides, where the eigenfrequencies  $\omega_j(k)$  in dependency of the quasi-momentum  $k \in B$  are searched in the Brillouin zone  $B$ . For photonic crystal wave-guides eigenvalue problem depends nonlinearly on the quasi-momentum  $k$  through Dirichlet-to-Neumann boundary conditions [2].

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## Structured eigenvalue backward errors of matrix pencils

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An  $n \times n$  regular matrix pencil  $L(z) = zA + B$  is said to be structured if  $(A, B)$  belong to a special subset of  $(\mathbb{C}^{n \times n})^2$ . Matrix pencils arising in most applications follow some structure and the use of structure-preserving algorithms is advisable for solving them. Structure preserving perturbation analysis is necessary to assess the accuracy of such algorithms. This involves computing backward errors with respect to structure preserving perturbations which we refer to as structured backward errors.

The structured eigenvalue backward errors are important for the stability analysis of structure preserving algorithms that compute only eigenvalues and for solving distance problems involving structured matrices [4]. Explicit formulas for structured eigenpair backward errors of matrix pencils have been developed in [1, 2, 3] for a number of important structures. However, structured eigenvalue backward errors have not been obtained in these works. Motivated by such considerations, explicit formulas for structured eigenvalue backward errors have been obtained for matrix pencils with Hermitian and related structures in [5] and for palindromic structures in [6] with respect to the norm  $\sqrt{\|A\|_2^2 + \|B\|_2^2}$  on  $L(z)$ , where  $\|\cdot\|_2$  is the matrix 2-norm.

In this talk, we briefly present some of the main results in [5] and [6], and focus on extensions to the case where the norm on  $L(z)$  is  $\max\{\|A\|_2, \|B\|_2\}$ . An important structure that often arises in applications is when the coefficient matrices of the pencil  $L(z)$  are real. Understanding the effect of real perturbations on real matrix pencils is a challenging task. The real eigenvalue and eigenpair backward errors are not known for real matrix pencils even when they have no additional structure. We present some results for structured eigenvalue/eigenpair backward errors of real structured pencils with respect to perturbations that preserve realness as well as additional symmetries. All the results are illustrated by numerical experiments.

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## Linearization schemes for Hermite matrix polynomials

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The polynomial eigenvalue problem is to find the eigenpair of  $(\lambda, x) \in \mathbb{C} \cup \{\infty\} \times \mathbb{C}^n \setminus \{0\}$  that satisfies  $P(\lambda)x = 0$ , where  $P(\lambda) = \sum_{i=0}^s P_i \lambda^i$  is an  $n \times n$  so-called matrix polynomial of degree  $s$ , where the coefficients  $P_i, i = 0, \dots, s$ , are  $n \times n$  constant matrices, and  $P_s$  is supposed to be nonzero. These eigenvalue problems arise from a variety of physical applications including acoustic structural coupled systems, fluid mechanics, multiple input multiple output systems in control theory, signal processing, and constrained least square problems. Most numerical approaches to solving such eigenvalue problems proceed by linearizing the matrix polynomial into a matrix pencil of larger size.

Such methods convert the eigenvalue problem into a well-studied linear eigenvalue problem, and meanwhile, exploit and preserve the structure and properties of the original eigenvalue problem. The linearizations have been extensively studied with respect to the basis that the matrix polynomial is expressed in. If the matrix polynomial is expressed in a special basis, then it is desirable that its linearization be also expressed in the same basis. The reason is due to the fact that changing the given basis ought to be avoided [3]. The authors in [1] have constructed linearization for different bases such as degree-graded ones (including monomial, Newton and Pochhammer basis), Bernstein and Lagrange basis. This contribution is concerned with polynomial eigenvalue problems in which the matrix polynomial is expressed in Hermite basis. In fact, Hermite basis is used for presenting matrix polynomials designed for matching a series of points and function derivatives at the prescribed nodes.

In the literature, the linearizations of matrix polynomials of degree  $s$ , expressed in Hermite basis, consist of matrix pencils with  $s + 2$  blocks of size  $n \times n$ . In other words, additional eigenvalues at infinity had to be introduced, see e.g. [2]. In this research, we try to overcome this difficulty by reducing the size of linearization. The reduction scheme presented will gradually reduce the linearization to its minimal size making use of ideas from [4]. More precisely, for  $n \times n$  matrix polynomials of degree  $s$ , we present linearizations of smaller size, consisting of  $s + 1$  and  $s$  blocks of  $n \times n$  matrices. The structure of the eigenvectors is also discussed.

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## Block Krylov subspace methods for shifted systems with different right-hand sides

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We present some new techniques for solving a family (or a sequence of families) of linear systems in which the coefficient matrices differ only by a scalar multiple of the identity (shifted systems). Our goal is to develop methods for shifted systems which have fewer restrictions usually associated with such methods (e.g., all residuals needing to be collinear).

The systems are parameterized by  $i$ ,

$$Ax = b \quad \text{and} \quad (A + \sigma_i I)x(\sigma_i) = b(\sigma_i), i = 1, 2, \dots, L, \quad (10)$$

with  $A \in \mathbb{C}^{n \times n}$  and  $\{\sigma_1, \dots, \sigma_L\} \subset \mathbb{C}$ . We can add a new parameter  $j$ , indexing a sequence of matrices  $\{A_j\} \subset \mathbb{C}^{n \times n}$ , and for each  $j$  we solve a family of systems

$$A_j x_j = b_j \quad \text{and} \quad (A_j + \sigma_{i,j} I)x(\sigma_{i,j}) = b(\sigma_{i,j}), i = 1, 2, \dots, L_j, \quad \text{where} \quad \{\sigma_{i,j}\}_{i=1}^{L_j} \subset \mathbb{C}. \quad (11)$$

Many methods have been proposed for solving (10) are built upon the invariance of Krylov subspaces under a scalar shift, i.e.,

$$\mathcal{K}_j(A, r_0) = \mathcal{K}_j(A + \sigma I, r_0(\sigma)) \quad (12)$$

which holds as long as the collinearity condition  $r_0(\sigma) = \beta_\sigma r_0$  is satisfied. This allows us to generate approximate solution corrections for all linear systems in (10) from the common Krylov subspace. These methods can be quite effective and allow for a great savings in both storage and computational costs. However, building methods on top of the invariance (12) introduces a severe restriction on the class of problems we can treat. Furthermore, we have shown that this restriction also hampers the integration of augmentation methods such as subspace recycling [2] into this setting in order to treat (11); see, [5]. Here we propose a technique which circumvent this problem while still taking advantage of the invariance (12). Block Krylov subspaces are shift invariant just as their single-vector counterparts. Thus by collecting all initial residuals into one block vector, we can generate a block Krylov subspace. Due to shift invariance, we can define block FOM- and GMRES-type projection methods to simultaneously solve all shifted systems. These are not block versions of the shifted FOM method [3] or the shifted GMRES method [1]. These methods are compatible with unrelated right-hand sides and residual collinearity is no longer a requirement at restart. Due to this special manner in which we take advantage of (12), subspace recycling may be integrated into the proposed methods in order to treat (11).

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## Damping optimization in mechanical systems with external force

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We consider a mechanical system excited by an external force. Model of such a system is described by the system of ordinary differential equations:  $M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = \hat{f}(t)$ , where matrices  $M, K$  (mass and stiffness) are positive definite and the vector  $\hat{f}$  corresponds to an external force. The damping matrix  $D$  is assumed to be positive semidefinite and has a small rank.

The motivation for our approach has been posted in [2, Section 17] and it is related to the harmonic response of the mechanical system under the influence of the harmonic force. Here we consider external function consisting of simple oscillating functions which is motivated by Fourier series which decomposes periodic functions into the sum of a set of simple oscillating functions. In this setting, we consider criterions average energy amplitude and average displacement amplitude that allow damping optimization of mechanical system excited by an external force.

Since in general a damping optimization is a very demanding problem, we provide a new explicit formulas which have been used for efficient damping optimization. The efficiency of new formulas has been illustrated with a numerical examples.

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## Finding a low-rank basis in a matrix subspace

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For a given matrix subspace, how can we find a basis that consists of low-rank matrices? This problem is a generalization of the sparse vector problem. When the subspace is spanned by rank-one matrices, a solution is equivalent to a tensor CP decomposition. If the information on the rank-one basis is not given in advance, or if the space is spanned by matrices of higher rank, the situation is not as straightforward. By standard arguments from matroid theory, the problem can be in theory solved using a greedy algorithm. In this work we present a practical algorithm that mimics this greedy procedure. It finds basis elements one by another in two stages, by first estimating a minimal rank by applying soft singular value thresholding to a nuclear norm relaxation, and then computing a matrix with that rank using the method of alternating projections. Given the hardness of the problem, our method provides surprisingly reliable results in a number of experiments. Potential applications include data compression beyond the classical truncated SVD, computation of “low-rank” eigenvectors to simple or even multiple eigenvalues, image separation, and others.

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# The linear-quadratic optimal control problem revisited

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One of themes that constantly appears in Volker's work is the linear-quadratic optimal control problem for linear time-invariant differential-algebraic equations, see, e.g., the monograph [1]. Volker and many other authors approached this problem by various techniques but to our best knowledge all of these put restrictive assumptions on the control system or the cost functional. In this talk we will discuss a new approach to overcome these restrictions by considering a Lur'e matrix equation on the system space, i.e., the space in which the solution trajectories of the system evolve. We will further study stabilizing and extremal solutions of this equation which can be used to construct optimal stabilizing feedbacks. These results can be interpreted as a generalization of the algebraic Riccati equation to a much larger class of linear-quadratic optimal control problems [2, 3].

If time permits we will discuss existence and uniqueness of the optimal control which can be characterized in terms of the zero dynamics of the closed-loop system.

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## Inexact nested Newton-ADI method to solve large-scale algebraic Riccati equations

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We investigate numerical methods to efficiently solve algebraic Riccati equations (ARE) like

$$\mathcal{R}(X) = C^T C + A^T X + X A - X B B^T X = 0 \quad (13)$$

with  $C \in \mathbb{R}^{p \times n}$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $X = X^T \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times r}$ ,  $p+r \ll n$ , by combining existing approaches. These quadratic matrix equations have to be solved, e.g., in optimal control problems to apply a *linear quadratic regulator* (LQR) approach [6, 7].

Our aim is an iterative solver for (13) based on the Newton-ADI method. Recent ADI improvements in [2, 3, 4] in combination with the inexact Kleinman-Newton approach in [5] and the line search method in [1] are the key ingredients to our novel approach that can handle large-scale problems efficiently. We show theoretical as well as numerical results that illustrate the usability of the novel approach as well as its advantages.

The open problem of controlling the accuracy of the solver for the shifted linear systems appearing in each ADI step will also be addressed shortly.

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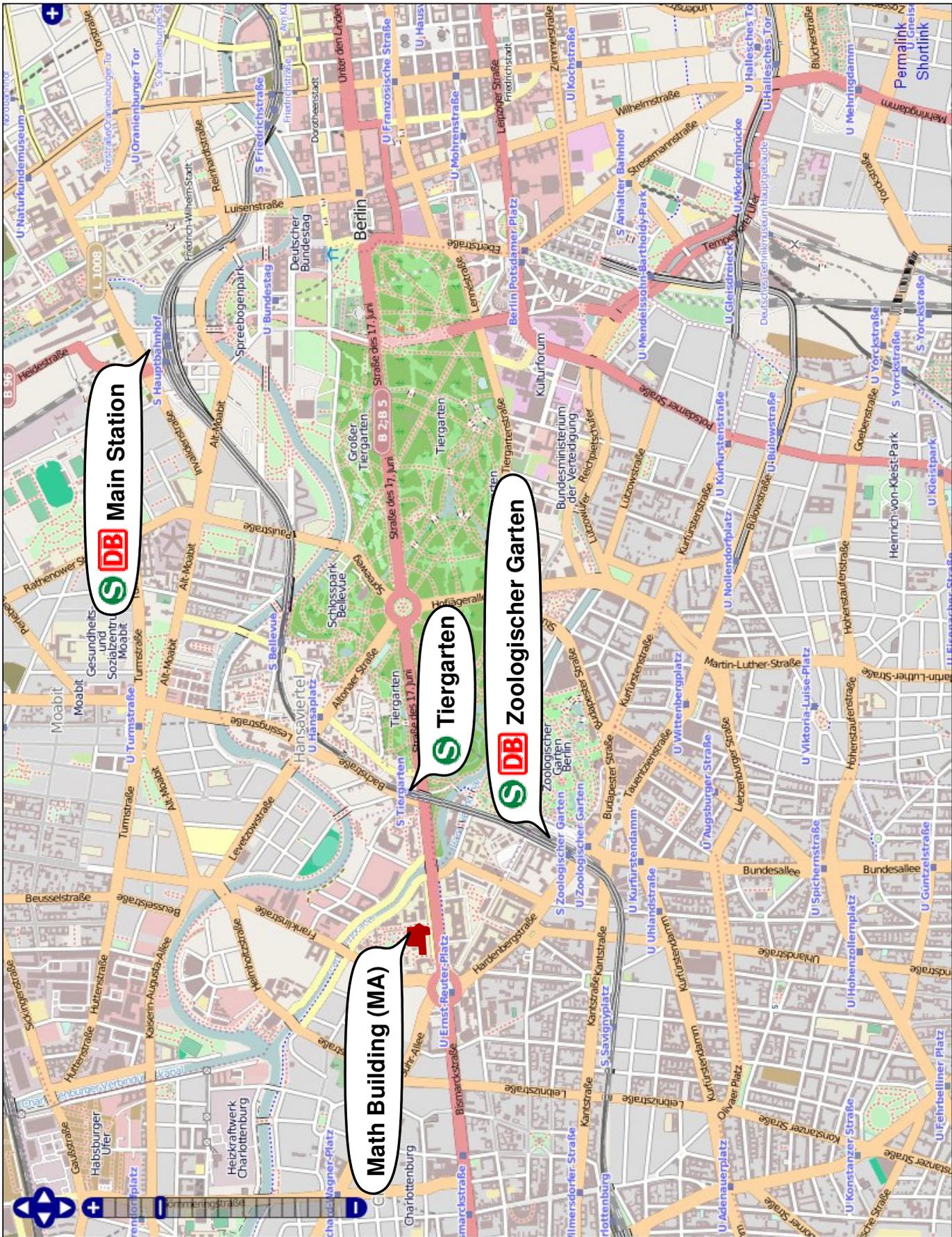
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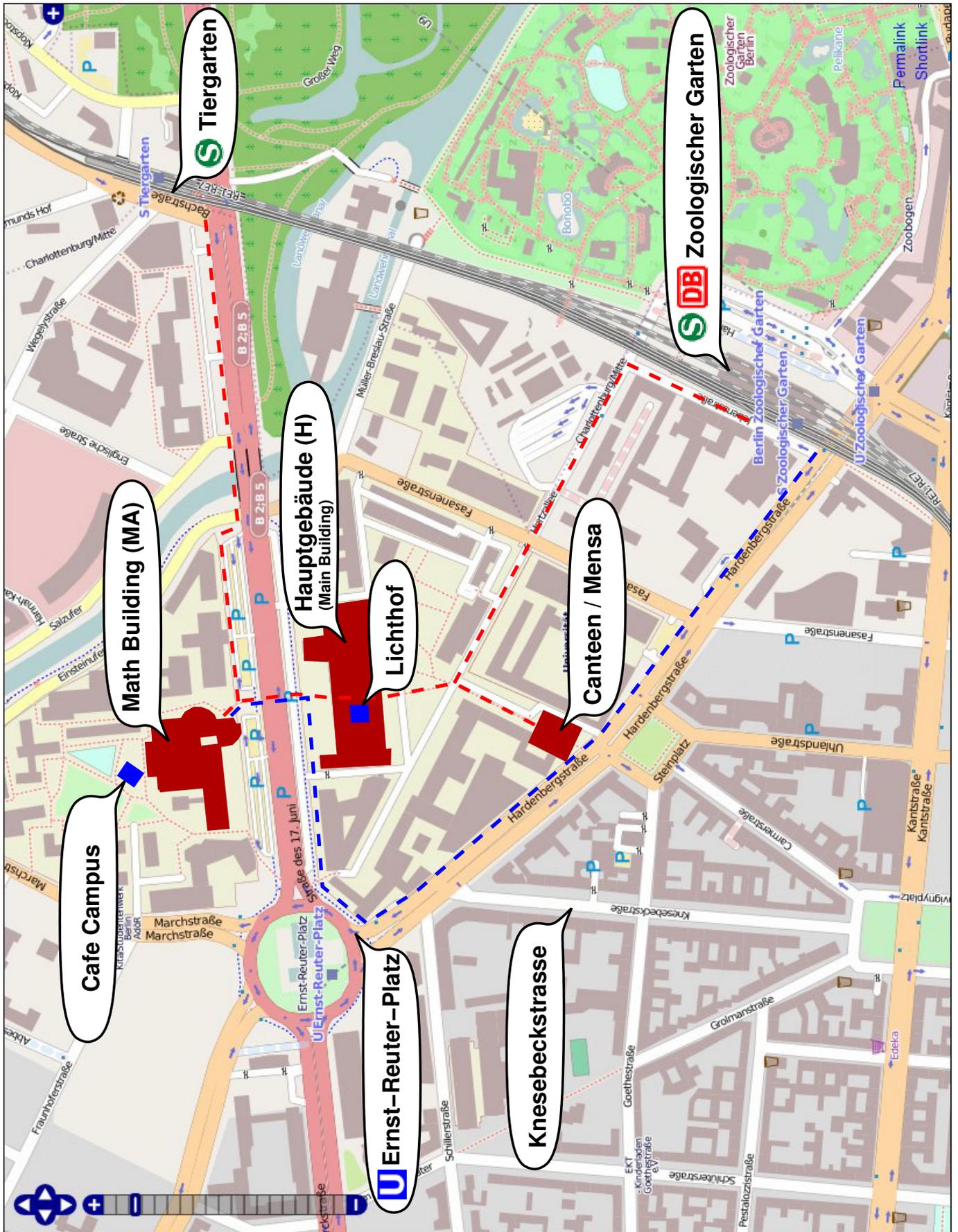
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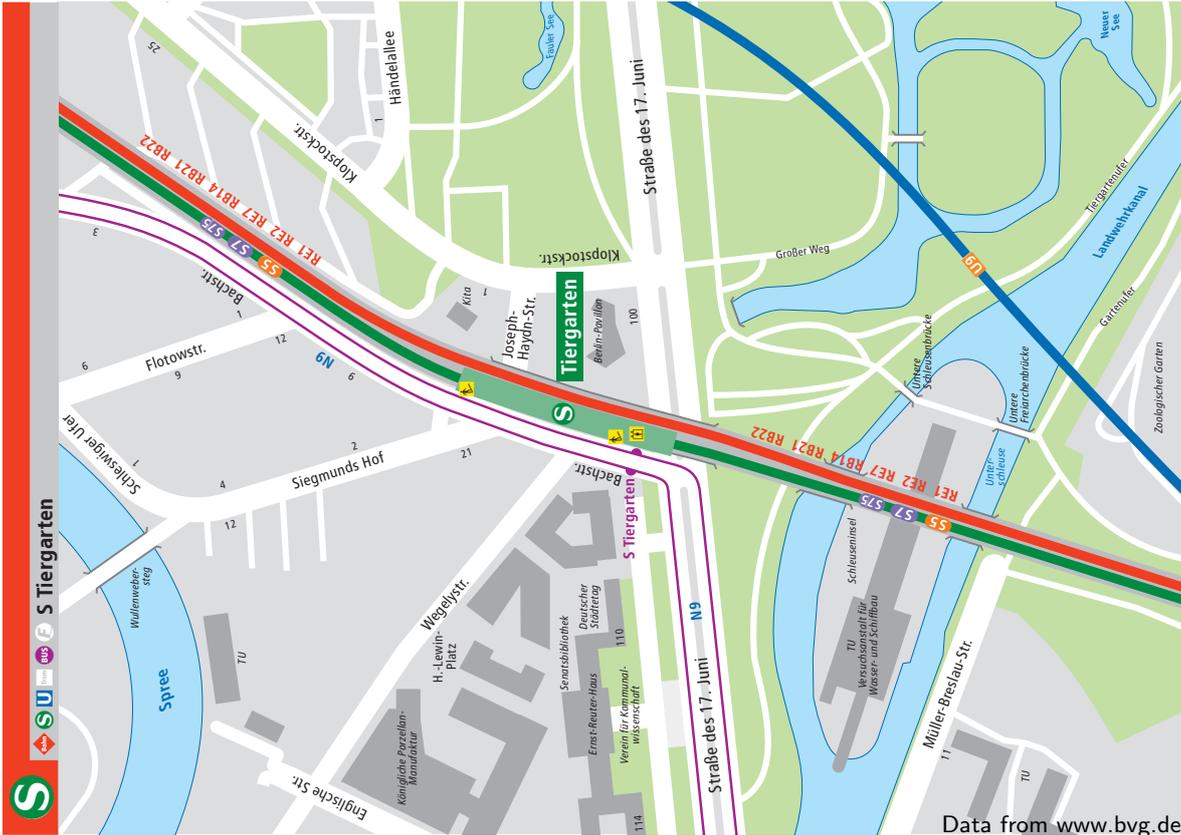
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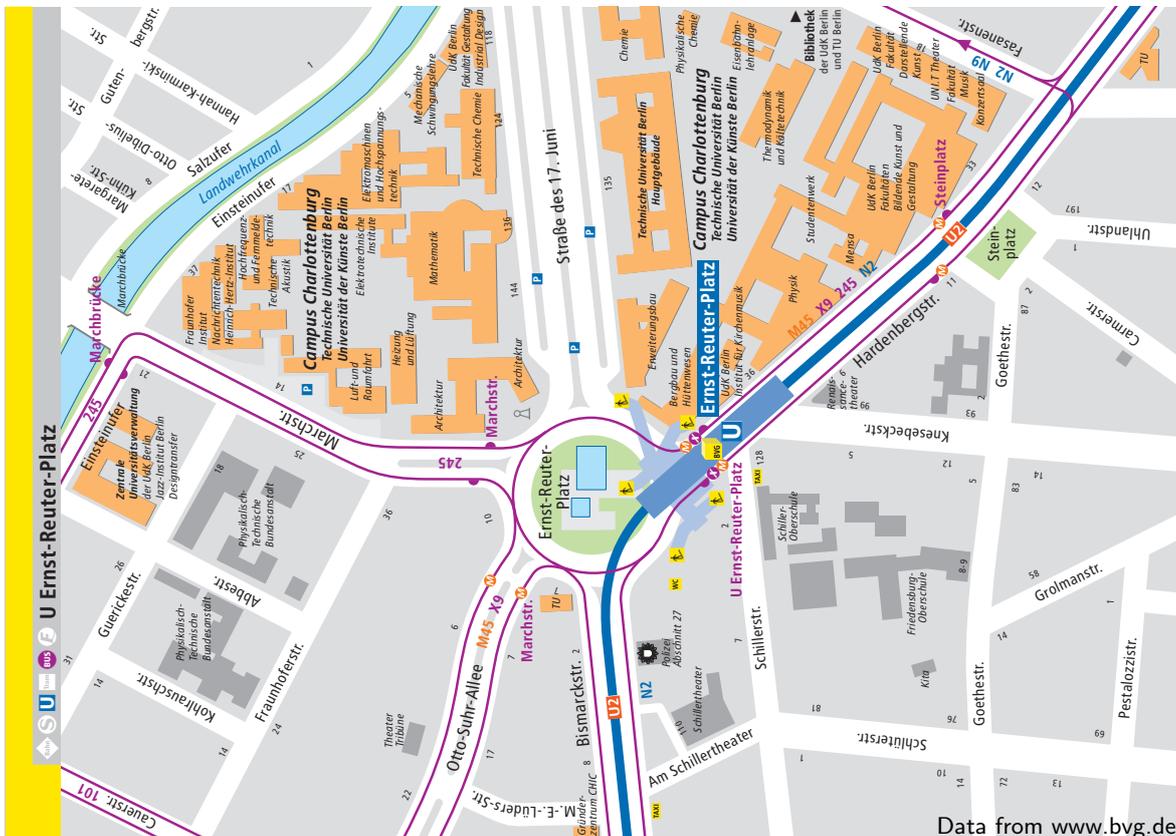
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