Co-Simulation and modal reduction for multifield problems in multibody dynamics

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Outline

1. Co-Simulation for weakly coupled problems
   • Co-Simulation and Modular time integration methods
   • Stability, Convergence, Overlapping methods
   • Dynamical interaction of multibody systems and large elastic structures

2. A modal multifield approach in multibody dynamics
   • Flexible multibody systems
   • Piezoelectricity, Thermoelasticity, Coupled model equations
   • Modal reduction for multifield problems
   • Thermal response modes
   • Industrial case study: Machine tool
Co-Simulation: Motivation

Simulation of heterogenous systems (system dynamics, micro systems, … )

\[
M(q)\ddot{q} = f(q, \dot{q}) - B^T(q)\lambda \\
0 = b(q)
\]

BDF, DAE’s, wheel/rail contact, …

\[
\rho A\dddot{w} = -EI\dddot{w} + Tw'' + \delta(x - x_p(t))\lambda
\]

FEM, modal reduction, Newmark, …

Data exchange at discrete synchronisation points $T_n$
Co-Simulation: Modular time integration

\[ M_1 \ddot{q}_1 = f_1(q_1, \dot{q}_1) - B_1^T \lambda \]
\[ M_2 \ddot{q}_2 = f_2(q_2, \dot{q}_2) - B_2^T \lambda \]
\[ 0 = B_1 a_1 + B_2 a_2 + \ldots \]

(a) Solve \( 1 \) with \( \lambda = \lambda_n \)
(b) Interpolation in \([T_n, T_{n+1}] \Rightarrow \bar{q}_1\)
(c) Solve \( 2^+ \) with \( q_1 = \bar{q}_1 \)
(d) New contact force \( \lambda = \lambda_{n+1} \)

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Modular time integration: Convergence

Extrapolation steps \( \longrightarrow \) modular time integration in part explicit

Deuflhard / Hairer / Zugck (1987):
- Convergence theory for half-explicit DAE time integration methods
- Contractivity condition to guarantee stability and convergence


- Subsystems: BDF, RK (stiffly accurate, implicit), error \( O(h_i^{k_i}) \)
- Extrapolation: from \([T_{n-r}, T_n] \) to \((T_n, T_{n+1})\), error \( O(H^{q+1}) \)
- Contractivity condition: \( L \alpha < 1/r \) with contractivity constant

\[ \alpha := \max \| (B_2 M_2^{-1} B_2^T)^{-1} (B_1 M_1^{-1} B_1^T) \| \]

\( \Rightarrow \) Global error of modular method \( O(h_1^{k_1} + h_2^{k_2} + H^{q+1}) \)

Mechanics (A. 1999), Circuit simulation (Günther 1999), Fluid/structure (Steindorf, Matthies 2001)
Overlapping modular time integration

Classical Gauss-Seidel  Extrapolation of $\lambda$
\[
M_1 \ddot{q}_1 = f_1(q_1, \dot{q}_1) - B_1^T \lambda \\
0 = b(q_1, q_2)
\]
\[
M_2 \ddot{q}_2 = f_2(q_2, \dot{q}_2) - B_2^T \lambda \\
0 = b(q_1, q_2)
\]

Stabilization  $\alpha = \max \| (B_2 M_2^{-1} B_2^T)^{-1} (B_1 M_1^{-1} B_1^T) \| < 1$

- Nonlinear projection steps (Kübler, Schiehlen 2000)
- Overrelaxation (A., Günther 1999)

Overlapping Gauss-Seidel  Extrapolation of $q_2, \lambda$
\[
M_1 \ddot{q}_1 = f_1(q_1, \dot{q}_1) - B_1^T \Lambda_1 \\
0 = b(q_1, q_2)
\]
\[
M_2 \ddot{q}_2 = f_2(q_2, \dot{q}_2) - B_2^T \Lambda_2 \\
0 = b(q_1, q_2)
\]
\[\alpha = 0\] possible with  $\lambda(t) = (I - A_1 - A_2) \lambda(t) + A_1 \Lambda_1(t) + A_2 \Lambda_2(t)$

Overlapping modular time integration (II)
\[
\begin{aligned}
\dot{y}_i &= \varphi_i(y_1, \ldots, y_r, w), \quad (i=1, \ldots, r) \\
0 &= \gamma(y_1, \ldots, y_r, w)
\end{aligned}
\]

Macro step
- Integrate $r$ subsystems separately with stage functions $Y_i, W_i$.
- Assign each constraint to $l \geq 1$ subsystems  $0 = P_i^T \gamma(y, w)$.
\[
\begin{aligned}
\dot{y}_i &= \varphi_i(Y_i, W_i) \\
0 &= P_i^T \gamma(Y_i, W_i)
\end{aligned}
\],  $(i=1, \ldots, r)$  $\Rightarrow$  $y_i, P_i^T W_i$

- Linear combination with weights $A_i$:
\[
w(t) = \left( I - \sum_{i=1}^r A_i P_i P_i^T \right) w(t) + \sum_{i=1}^r A_i P_i P_i^T W_i(t)
\]

Theorem  There are weights $A_i(t)$ such that the contractivity condition is satisfied with $\alpha = 0$. 

Martin Arnold, Andreas Heckmann: Multifield problems in multibody dynamics
7th Workshop on Descriptor Systems, Paderborn, March 2005.
Co-Simulation: Passing manoeuvre on bridge

Data exchange (sampling interval 1.0 ms)
- Vertical displacements
- Contact forces

CPU-time (PIII, 500 MHz): 580.0 s

2. Modal multifield approach in multibody dynamics

Elements (Rigid and flexible) bodies, joints, forces, ...

Model equations Principles of classical mechanics

\[ M(q)\ddot{q} = f(q, \dot{q}) - B^T(q)\lambda \]
with \( B(q) := \frac{\partial b}{\partial q}(q) \)

with (redundant) position coordinates \( q \)
Flexible multibody systems: Modal reduction

- **FEM Model**
  - FEM Results
    - Mass matrices
    - Stiffness matrices
    - Mode matrices

- **Modal reduction**
  - Volume integrals

- **User**
  - Dynamic loads

- **FATIGUE**
  - Covariance
  - Rainflow

- **FEMBS**
  - Modal reduction

- **MBS Model**

- **MBS Results**

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Multifield problems: Thermoelasticity

Coupling of elastic deformation and temperature field by loads / heat sources

\[
M_r \ddot{q}(t) = f_r(q, \dot{q}, u, \dot{u}) \\
\rho \ddot{\dot{u}}(x, t) = \ldots + f_u(u, \dot{u}, \dot{\dot{u}}) \\
\dot{\phi}(x, t) = \ldots + f_\phi(q, \dot{q}, \dot{\phi})
\]

- **Example 1** Hot spot scenario

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**Multifield problems: Thermoelasticity (II)**

Coupling of elastic deformation and temperature field by loads / heat sources

\[
\begin{align*}
M_r \ddot{q}(t) &= f_r(q, \dot{q}, u, \dot{u}) \\
\varrho \ddot{u}(x, t) &= \ldots + f_u(u, \dot{u}, \dot{\vartheta}) \\
\dot{\vartheta}(x, t) &= \ldots + f_\vartheta(q, \dot{q}, \dot{\vartheta})
\end{align*}
\]

**Example 2** Disc with thermal loads

- 2D model
- Transient temperature field with time independent boundary conditions
- Neumann condition at right boundary
- Robin condition at left boundary

**Multifield problem: Linear Material Constitution**

G. Heckmann (1925)

\[
\begin{pmatrix}
\sigma \\
d \\
\eta
\end{pmatrix}
= \begin{pmatrix}
H_\varepsilon & -H_\varepsilon^T & -H_\lambda^T \\
H_\varepsilon & H_\varepsilon & H_p \\
H_\lambda & H_p^T & H_\alpha
\end{pmatrix}
\begin{pmatrix}
\varepsilon \\
\vartheta
\end{pmatrix}
\]
Modal multifield approach: Model equations

\[
\begin{pmatrix}
M_{uu} & M_{u\alpha} & M_{u\varphi} \\
M_{u\alpha} & M_{\alpha\alpha} & M_{u\alpha} \\
M_{u\varphi} & M_{u\alpha} & M_{\varphi\varphi}
\end{pmatrix}
\begin{pmatrix}
a_h \\
a_{\alpha} \\
\ddot{u}
\end{pmatrix}
= 
\begin{pmatrix}
h_u \\
h_{\alpha} \\
h_{\varphi}
\end{pmatrix}
- 
\begin{pmatrix}
0 \\
0 \\
K_{uu} + K_{u\alpha} + K_{u\varphi}
\end{pmatrix}
\begin{pmatrix}
\ddot{u} \\
\ddot{\alpha} \\
\ddot{\varphi}
\end{pmatrix}
\]

\[
K_{uu} = \int B_u^T H_c B_u \, dV \\
K_{u\alpha} = \int B_u^T \Phi^T \Phi \, dV \\
K_{u\varphi} = \int B_u^T H_e^T B_{\varphi} \, dV
\]

Thermal response modes

Finite element discretization of multifield equations

\[
M_{uu} \ddot{z}_u + D_{uu} \dot{z}_u + K_{uu} z_u = \ldots + h_u(z_u, \dot{z}_u, \ddot{z}_u)
\]

\[
C_{\theta\theta} \ddot{z}_\theta + K_{\theta\theta} z_\theta = \ldots + h_\theta(q, \dot{q}, z_\theta)
\]

Modal approach

1. Deformation field: Eigenmodes \((M_{uu} \lambda_j^2 + D_{uu} \lambda_j + K_{uu}) z_u^{(j)} = 0\),
   static modes \((M_{uu} \lambda_j^2 + D_{uu} \lambda_j + K_{uu}) z_u^{(j)} = h_u^{(j)}\), etc.

2. Temperature field: Thermal modes \((C_{\theta\theta} \kappa_i + K_{\theta\theta}) z_\theta^{(i)} = h_\theta^{(i)}\).

3. Coupling in multifield problem: Thermal response modes resulting from a static analysis \(K_{uu} z_u^{(i)} = h_u(\ldots, z_\theta^{(i)})\).
### Thermal response modes: Disc with thermal loads

**Comparison FEM vs. MMA**

\[ u_z(x_{101}, t), \ \vartheta(x_{101}, t) \]

**Thermal and thermal response modes**

1. [Image of temperature distribution 1]
2. [Image of temperature distribution 2]
3. [Image of temperature distribution 3]
4. [Image of temperature distribution 4]
5. [Image of temperature distribution 5]
6. [Image of temperature distribution 6]
7. [Image of temperature distribution 7]

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**Industrial application: Machine tool**

- High speed: mounting of &gt; 50,000 elements per hour
- Acceleration at tool center point: 4 g
- High precision: tolerances &lt;100µm

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Industrial application: Machine tool (II)

Machine tool: Thermal response modes

- Long-term simulation with kinematics pre-defined by periodic input function
- Thermoelastic coupling defines quasi-stationary input $q(y)$ for heat equation
Machine tool: Thermal response modes (II)

Quasi-stationary temperature field

2 Steady state heat transfer FEM-solutions
⇒ 2 Thermal modes
⇒ 2 Thermal response modes

Machine tool: Thermal response modes (III)

Comparison FEM vs. MBS

Eigenvalue Analysis

Mechanical eigenmodes
\{ 90 Hz, 105 Hz, 114 Hz, ... 384 Hz, 387 Hz, 391 Hz \}

Thermal response modes
\{ 1694 Hz, 2368 Hz \}
Machine tool: Dynamical simulation

TCP: Tool center point
P2: Reference, moving along the machine base with pre-defined kinematics
$\Delta r_3$, $\Delta z_3$: Reference displacements of TCP without thermal loads

Summary

Simulation of coupled instationary problems in multibody dynamics

Co-Simulation: Coupling of simulation tools

- Modular time integration methods: Stability, Convergence
- Stabilization by overlapping techniques
- Application to weakly coupled problems

Modal reduction for multifield problems

- Coupled model equations: Piezoelectricity, Thermoelasticity
- Modal reduction: Thermal response modes
- Industrial case study: Machine tool

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