High-Order Quasi Monte-Carlo for Bayesian Inversion in High Dimension

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Abstract

We consider the problem of Bayesian Inverse Problems for partial differential equations with "distributed random input". This is to say that the uncertain input data take values in function space X. Upon parametrization of these inputs in terms of a basis of the function space X, the problem of Bayesian estimation is converted to the problem of numerical evaluation of high-dimensional integrals, with infinite-dimensional, parametric PDEs as forward problems.

We establish, for a broad class of elliptic and parabolic inverse problems, and for inversion with respect to domains, coefficients and forcing functions, that the associated integrand functions belong to a class of weighted function spaces on the high-dimensional parameter domain.

The regularity estimates justify higher order (single and multi-level) Quasi Monte-Carlo algorithms for the efficient numerical Bayesian estimation.

Work supported in part by the Swiss National Science Foundation under grant No. 149819 and by CPU time from the Swiss National Computing Center under CSCS Project ID d41.

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