Abstracts
Winter School on Compressed Sensing

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Massimo Fornasier

Let us assume that f is a continuous k-ridge function defined on the unit ball of $\mathbb{R}^d$, of the form $f(x) = g(Ax)$, where A is a $k \times d$ matrix and g is a function of k variables for $k \ll d$. We are given a budget $m \in \mathbb{N}$ of possible point evaluations $f(x_i)$, $i = 1, \ldots, m$, of f, which we are allowed to query in order to construct a uniform approximating function. Under certain smoothness and variation assumptions on the function g, and an *arbitrary* choice of the matrix A, we present in this the first lecture 1. a sampling choice of the points $\{x_i\}$ drawn at random for each function approximation; 2. algorithms for computing the approximating function, whose complexity is at most polynomial in the dimension d and in the number m of points. Due to the arbitrariness of A, the choice of the sampling points will be according to suitable random distributions and our results hold with overwhelming probability. Our approach uses tools taken from the *compressed sensing* framework, recent Chernoff bounds for sums of positive-semidefinite matrices, and classical stability bounds for invariant subspaces of singular value decompositions.

In the second lecture we address the uniform approximation of sums of ridge functions $\sum_{i=1}^{m} g_i(a_i \cdot x)$ on $\mathbb{R}^d$ from a small number of query samples, under mild smoothness assumptions on the functions g_i's and near-orthogonality of the ridge directions a_i's. The sample points are again randomly generated and are *universal*, in the sense that the sampled queries on those points will allow the proposed recovery algorithms to perform a uniform approximation of any sum of ridge function with high-probability. Our general approximation strategy is developed as a sequence of algorithms to perform individual sub-tasks. We first approximate the span of the ridge directions. Then we use a straightforward substitution, which reduces the dimensionality of the problem from d to m. The core of the algorithm is the approximation of ridge directions expressed in terms of rank 1 matrices $a_i \otimes a_i$, realized by formulating their individual identification as a suitable nonlinear program, maximizing the spectral norm of certain competitors constrained over the unit Frobenius sphere. The final step is then to approximate the functions $g_1, \ldots, g_m$ by $\hat{g}_1, \ldots, \hat{g}_m$.

Gitta Kutyniok

Compressed sensing is the idea that it should be possible to capture attributes of a signal using very few measurements. Since publication of the initial papers in 2006, it has already become a key concept in various areas of applied mathematics, computer science, and electrical engineering. The mathematical foundations are nowadays quite well understood. Key to compressed sensing is the surprising fact that high-dimensional signals, which allow a sparse representation by a suitable basis or, more generally, a frame, can be recovered from very few linear measurements by using efficient algorithms such as convex optimization approaches.

In the first lecture, we will give an introduction to this vivid and highly interdisciplinary research area. The other lectures will discuss specific representation systems from applied harmonic analysis, which provide such sparse representations, in particular, for images, and their utilization for compressed sensing methods. A special focus will be on shearlet frames, which are optimally suited to provide sparse representations of anisotropic features such as edges in images. We then apply these ideas to the problems of data separation, recovery of missing data, and sampling of Fourier measurements. For each of those, we will provide a basic understanding, explain the methodological approach, discuss its theoretical analysis, and present numerical experiments.

Holger Rauhut

Compressed sensing predicts that sparse vectors can reconstructed from highly incomplete linear measurements via efficient algorithms such as ℓ_1-minimization. Provably optimal result on the required number of measurements in terms of the sparsity and signal length are known for random measurement matrices. This can be shown in a relatively simple way for Gaussian random matrices where all entries are independent standard normal distributed. Practical applications, however, lead to much more structure in the measurement matrix and there is only limited randomness can be injected into the measurement process. Such considerations lead to the study of structured random matrices. Several constructions will be reviewed in the lectures. One important case is the recovery of sparse vectors from few randomly selected Fourier coefficients. Recovery results including proof ideas for the corresponding random partial Fourier matrix will be presented. A second important construction deals with the recovery of sparse signals from a subsampled convolution with a random vector. The analysis of this scenario involves recent bounds on the supremum of so-called chaos processes via chaining methods. A few applications of structured random measurements in compressed sensing will be discussed as well.

Roman Vershynin

The classical problem of compressed sensing is to recover a signal from few random linear measurements. This problem has been well studied in the last decade. But what if the measurements are not linear? Binary measurements are an important example; they arise in various classification problems (e.g. sick/healthy). In other examples, the nature of non-linearity could even be unknown. Fortunately, compressed sensing could still be applied for non-linear measurements. We will theoretically validate the following heuristic. Non-linear measurements may be treated as noisy linear measurements, and thus the signal should be recoverable using linear methods such as Lasso. The proof of this heuristic is a good illustration of methods of high dimensional probability. It is based on recent work of Yaniv Plan and the speaker.

Rachel Ward

An important extension of compressive sensing is to the setting of low-rank matrix recovery from undersampled linear measurements. We first recall that an n by n matrix of rank-r can often be exactly recovered via convex optimization from a set of $\mathcal{O}(nr)$ observed linear measurements, provided the measurements are sufficiently generic. This result has quite strong implications as $\mathcal{O}(nr)$ measurements is much smaller than the ambient matrix size when the rank is small. Still, the underlying semidefinite programming routines become quite expensive even for moderately large n, and it is of interest to derive more efficient algorithms with comparable recovery guarantees. We discuss several recent results of this type for local gradient descent algorithms, and if time permits, finish by discussing open directions and extensions of such algorithms.

Applications

Axel Flinth

In wireless communications, a number of users are transmitting sequences of bits by sending electromagnetical waves through the air to a central, which has a certain number of antennas m. Typically, the total number of sending antennas d is much larger than m. Since only a few users are transmitting at every given moment, the collection of transmitted signals is sparse, and therefore, compressed sensing methods can be applied to recover the signal x_0 from the incoming waves.

An important algorithm for sparse recovery is *Orthogonal Matching Pursuit* (OMP). An empirical truth is that OMP is fast, at least when the sparsity is low, but has worse recovery probabilites than e.g. Basis Pursuit. It also gets slow as the sparsity level of the signal x_0 gets larger. In our specific application, the signals are however not only sparse but also integer-valued. Can we modify OMP in such a way that its recovery performance and/or speed increases for this special class of signals?

This talk will present the new algorithm $PROMP$, which uses an ℓ_2-minimization step to speed up OMP. We will conduct a theoretical performance analysis for the case that the measurement matrix is Gaussian. Additionally, some small numerical experiments will be discussed.

Martin Genzel

Tumor diseases, such as cancer, rank among the most frequent causes of death in Western countries. The clinical research of the last decades has shown that the pathological mechanisms of many diseases are manifested on the level of *protein activities*. In order to improve the clinical *treatment options* and *early diagnostics*, it is therefore necessary to better understand protein structures and their interactions. The related research field of *proteomics* focuses on analyzing the so-called *proteome*, which denotes the entire set of proteins of a human individual at a certain point of time. Unfortunately, proteomics-data, e.g., produced by *mass spectrometry*, is usually extremely *high-dimensional*. Therefore, it is a very difficult task to extract a *disease fingerprint*, which is a small set of proteins allowing for an appropriate classification of a patient's health status.

In the first part of this talk, we will see that the assumption of *sparsity* can help us to cope with this challenge. In this context, the method of *Sparse Proteomics Analysis* (SPA) will be introduced, enabling us to build sparse and reliable classifiers. The second part of the talk is then devoted to a theoretical foundation of SPA. Relying on a simple linear *forward model* for the data, we will see that very recent results from *high-dimensional estimation theory* can be used to prove rigorous recovery guarantees.

Jackie Ma

In this talk we discuss the application of compressed sensing and shearlets in magnetic resonance imaging (MRI) and electron microscopy (EM). Indeed, it is of utmost importance to keep the acquisition time as short as possible since, for instance, in MRI patients are not allowed to move during the measurement process. However, also the image quality should not decrease dramatically otherwise important desease features may get lost. Hence, in this talk we will explain the sampling model and the sparsity scheme in these applications that allow a significant undersampling during the acquisition of data while still allowing image reconstructions of sufficiently high quality.