Compressed Sensing and High-Resolution Image Inversion

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Image Inversion

\[ y(t) = \sum_{i=1}^{k} \psi(\nu_i) \theta_i \quad \rightarrow \quad \{y(t_i)\}_{i=1}^{m} \quad \rightarrow \quad \{\hat{\nu}_i, \hat{\theta}_i\}_{i=1}^{k} \]
Classical Inversion vs Compressed Sensing

- **Classical: Matched filtering**
  - Sequence of rank-one subspaces, or one-dimensional test images, is matched to the measured image by filtering or correlating or phasing.
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  - Extends to subspace matching for those cases in which the model for the image is comprised of several dominant modes.
  - Extends to whitened matched filter, or minimum variance unbiased (MVUB) filter, or generalized sidelobe canceller.
Classical Inversion vs Compressed Sensing

- **Classical: Estimation in Separable Model**
  - Low-order separable modal representation for the field.
  - Estimates of linear parameters (complex amplitudes of modes) and nonlinear mode parameters (frequency, wavenumber, delay, and/or doppler) are extracted, usually based on maximum likelihood, or some variation on linear prediction, using $l_2$ minimization.
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  - SNR, Fisher Information, Cramer-Rao Bound (CRB), Kullback-Leibler divergence, Bayesian CRB, Threshold Effects.
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- **Sampling**: Any subsampling of the measured image has consequences for resolution (or bias) and for variability (or variance).
Compressed Sensing:

- Subsampling has manageable consequences for image inversion provided we have known sparsity structure.
- Typically employs randomly drawn linear combinations.
1. **Fisher Information**: What is the impact of compressive sampling on Fisher information and Cramer-Rao bound (CRB) for estimating nonlinear parameters?
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2. **Breakdown Thresholds**: What is the impact of compressive sampling on SNR thresholds at which mean-squared error in estimating parameters deviate sharply from the CRB?
1. **Fisher Information:** What is the impact of compressive sampling on Fisher information and Cramer-Rao bound (CRB) for estimating nonlinear parameters?

2. **Breakdown Thresholds:** What is the impact of compressive sampling on SNR thresholds at which mean-squared error in estimating parameters deviate sharply from the CRB?

3. **Model mismatch:** What is the sensitivity of compressed sensing to model mismatch? Can these sensitivities be mitigated?
Fisher Information and Cramer-Rao Bound
Fisher Information and Cramer-Rao Bound

- **Fisher Information:**

\[ J(\theta) = E\left[ \left( \frac{\partial \log f(y; \theta)}{\partial \theta} \right) \left( \frac{\partial \log f(y; \theta)}{\partial \theta} \right)^H \right]. \]

- **Cramer-Rao Bound:** The inverse \( J^{-1}(\theta) \) lower bounds the error covariance matrix for any unbiased estimator of \( \theta \).
Complex Normal model:
\[ y = s(\theta) + n \in \mathbb{C}^n; \quad y = \mathcal{CN}_n[s(\theta), R] \]

Fisher information matrix:
\[ J(\theta) = G^H(\theta)R^{-1}G(\theta) \]
\[ = \frac{1}{\sigma^2} G^H(\theta)G(\theta), \quad \text{when} \quad R = \sigma^2 I \]
\[ G(\theta) = [g_1(\theta), \ldots, g_k(\theta)]; \quad g_i(\theta) = \frac{\partial s(\theta)}{\partial \theta_i} \]

Cramer-Rao lower bound:
\[ (J^{-1}(\theta))_{ii} = \sigma^2 (g_i^T(\theta)(I - P_{G_i(\theta)}))g_i(\theta))^{-1} \]

When one sensitivity looks like a linear combination of others, performance is poor.
CS, Fisher Information, and CRB

- **Compressive measurement:**
  \[ z = \Phi y = \Phi [s(\theta) + n] \in \mathbb{C}^m; \]

- **Fisher information matrix**
  \[ \hat{J}(\theta) = \frac{1}{\sigma^2} G^H(\theta) P_{\Phi^H} G(\nu) = \hat{G}^H(\theta) \hat{G}(\theta) \]

- **Fisher information matrix**
  \[ \hat{G}(\theta) = [\hat{g}_1(\theta), \ldots, \hat{g}_k(\theta)]; \quad \hat{g}_i(\theta) = P_{\Phi^H} \frac{\partial s(\theta)}{\partial \theta_i} \]

- **Cramer-Rao lower bound:**
  \[ (\hat{J}^{-1}(\theta))_{ii} = \sigma^2 (\hat{g}_i^T(\theta)(I-P_{\hat{G}_i(\theta)})\hat{g}_i(\theta))^{-1} \]

Compressive measurement reduces the distance between subspaces: loss of information.
Question: What is the impact of compressive sampling on the Fisher information matrix, Cramer-Rao bound (CRB), and Kullback-Leibler divergence for estimating parameters?
**JL Lemma:** For any $\epsilon \in (0, 1)$, a random linear transformation $\Phi : \mathbb{R}^n \to \mathbb{R}^m$ is said to satisfy an $\epsilon$–JL type Lemma over a set of vectors $Q \subset \mathbb{R}^n$ with probability at least $1 - \delta$ if

$$\Pr \left( \forall q \in Q : (1 - \epsilon)\|q\|_2^2 \leq \|\Phi q\|_2^2 \leq (1 + \epsilon)\|q\|_2^2 \right) \geq 1 - \delta.$$ 

For random matrices with i.i.d. $\mathcal{N}(0, 1/m)$ entries $\Phi_{ij}$, we have $\delta \leq 2|Q|e^{-mc_0(\epsilon)}$ where $c_0(\epsilon) = \epsilon^2/4 - \epsilon^3/6$ [Baraniuk, Davenport, Devore, and Wakin ’08; Dasgupta and Gupta ’02].
**Subspace JL Lemma:** [Sarlos '06] Let $\Phi : \mathbb{R}^n \to \mathbb{R}^m$, $m < n$, and $\epsilon \in (0, 1)$. Then $\Phi$ satisfies the $\epsilon$–JL type Lemma over any arbitrary $p$-dimensional subspace $\langle \mathbf{V} \rangle$ of $\mathbb{R}^n$ with probability at least $1 - \delta$, provided that it satisfies the $\epsilon'–$JL type Lemma over any set $Q \subset \mathbb{R}^n$ of $\lceil (2\sqrt{p}/\epsilon')^p \rceil$ vectors with probability at least $1 - \delta$, where $\epsilon'$ satisfies

$$
\left( \frac{3\epsilon'}{1 - \epsilon'} \right)^2 + 2\left( \frac{3\epsilon'}{1 - \epsilon'} \right) = \epsilon.
$$
**Theorem:** [Pakrooh, P., Scharf, Chi '13]

(a) For any compression matrix, we have

\[(J^{-1}(\theta))_{ii} \leq (\hat{J}^{-1}(\theta))_{ii} \leq 1/\lambda_{\text{min}}(G^T(\theta)P_{\Phi^T}G(\theta))\]

(b) For a random compression matrix, we have

\[(\hat{J}^{-1}(\theta))_{ii} \leq \frac{\lambda_{\text{max}}(J^{-1}(\theta))}{\lambda_{\text{min}}((\Phi\Phi^T)^{-1})} \leq \frac{\lambda_{\text{max}}(J^{-1}(\theta))}{C(1-\epsilon)}\]

with probability at least \(1 - \delta - \delta'\), where

- \(1 - \delta\) is the lower bound on the probability that \(\Phi\) satisfies the \(\epsilon\)-JL type lemma for any \(p\)-dimensional subspace, and
- \(1 - \delta'\) is the probability that \(\lambda_{\text{min}}((\Phi\Phi^T)^{-1})\) is larger than \(C\).
For tr($\hat{J}^{-1}(\theta)$) we have

$$\text{tr}(J^{-1}(\theta)) \leq \text{tr}(\hat{J}^{-1}(\theta)) \leq \frac{p\lambda_{\text{max}}(J^{-1}(\theta))}{C(1 - \epsilon)}$$

where again the upper bound holds with probability at least $1 - \delta - \delta'$. 

We can also bound det($\hat{J}^{-1}(\theta)$).
CRB after Compression

Bounds on the CRB for $-2\pi/n \leq \theta_2 \leq 2\pi/n$, $m = 3000, n = 8192$

Upper bounds on $\delta$ versus the number of measurements $m$ for $n = 8192$ and $\epsilon = 0.66$ (red) and $\epsilon = 0.33$ (green)
CS and Kullback-Leibler Divergence

**KL divergence** between $\mathcal{N}(x(\theta), R)$ and $\mathcal{N}(x(\theta'), R)$:

$$D(\theta, \theta') = \frac{1}{2}[(x(\theta) - x(\theta'))^T R^{-1}(x(\theta) - x(\theta'))].$$

- After compression with $\Phi$:

$$\hat{D}(\theta, \theta') = \frac{1}{2}[(x(\theta) - x(\theta'))^T \Phi^T (\Phi R \Phi^T)^{-1} \Phi (x(\theta) - x(\theta'))].$$

- With white noise $R = \sigma^2 I$:

$$\hat{D}(\theta, \theta') = \frac{1}{2\sigma^2}[(x(\theta) - x(\theta'))^T P_{\Phi^T} (x(\theta) - x(\theta'))].$$

**Theorem:** [Pakrooh, P., Scharf, and Chi (ICASSP'13)]

$$C(1 - \epsilon) D(\theta, \theta') \leq \hat{D}(\theta, \theta') \leq D(\theta, \theta')$$

with probability at least $1 - \delta - \delta'$, where $\delta$, $\delta'$. 


Nielsen, Christensen, and Jensen (ICASSP’12): Bounds on mean value of Fisher Information after random compression.

Ramasamy, Venkateswaran, and Madhow (Asilomar’12): Bounds on Fisher information after compression in a different noisy model.

Breakdown Threshold and Subspace Swaps
Breakdown Threshold and Subspace Swaps

- **Threshold effect:** Sharp deviation of Mean Squared Error (MSE) performance from Cramer-Rao Bound (CRB).

- **Breakdown threshold:** SNR at which a threshold effect occurs with non-negligible probability.

Donald W. Tufts (1933-2012)
Subspace Swap: Event in which measured data is more accurately resolved by one or more modes of an orthogonal subspace to the signal subspace.

- Cares only about what the data itself is saying.
- Bound probability of a subspace swap to predict breakdown SNRs.
Before compression:

$$y : \mathcal{CN}_n[\text{Hu}, \sigma^2 I]$$

After compression with left-orthogonal $\Phi \in \mathbb{C}^{m \times n}, m < n$:

$$y : \mathcal{CN}_m[\text{Gu}, \sigma^2 I], \ G = \Phi H$$
Before compression:

\[ y : \mathcal{CN}_n[0, HR_{uu}H^H + \sigma^2 I] \]

After compression with left-orthogonal \( \Phi \in \mathbb{C}^{m \times n}, m < n \):

\[ y : \mathcal{CN}_m[0, GR_{uu}G^H + \sigma^2 I], \quad G = \Phi H \]

Assume data consists of \( L \) iid realizations of \( y \) arranged as \( Y = [y_1, y_2, \cdots, y_L] \).
Subspace Swap Events

- **Subspace Swap Event** $E$: One or more modes of the orthogonal subspace $\langle A \rangle$ resolves more energy than one or more modes of the noise-free signal subspace $\langle H \rangle$. 

![Diagram showing Subspace Swap Events](image)
Subspace Swap Events

- **Subevent F:** Average energy resolved in the orthogonal subspace $\langle A \rangle$ is greater than the average energy resolved in the noise-free signal subspace $\langle H \rangle$.

  $$
  \min_i |h_i^H y|^2 \leq \frac{1}{p} y^H P_H y < \frac{1}{n-p} y^H P_A y \leq \max_i |a_i^H y|^2
  $$

- **Subevent G:** Energy resolved in the apriori minimum mode $h_{min}$ of the noise-free signal subspace $\langle H \rangle$ is smaller than the average energy resolved in the orthogonal subspace $\langle A \rangle$.

  $$
  |h_{min}^H y|^2 < \frac{1}{n-p} y^H P_A y \leq \max_i |a_i^H y|^2.
  $$
**Theorem:** [Pakrooh, P., Scharf (GlobalSIP’13)]

1. **Before compression:**

   \[ P_{ss} \geq 1 - P \left[ \frac{y^H P_H y}{y^H P_A y / (n - p)} > 1 \right] \]

   \[ = 1 - P \left[ F_{2p,2(n-p)} \left( \frac{\|H u\|_2^2}{\sigma^2} \right) > 1 \right] \]

   \( \|H u\|_2^2/\sigma^2 \) is the SNR before compression.

2. **After compression:**

   \[ P_{ss} \geq 1 - P \left[ F_{2p,2(m-p)} \left( \frac{\|G u\|_2^2}{\sigma^2} \right) > 1 \right] \]

   \( \|G u\|_2^2/\sigma^2 \) is the SNR after compression, \( G = \Phi H \).
Theorem: [Pakrooh, P., Scharf (GlobalSIP’13)]

- (a) Before compression:

\[ P_{ss} \geq 1 - P \left[ \frac{tr(Y^H P_H Y / pL)}{tr(Y^H P_A Y / (n - p)L)} > 1 \right] \]
\[ = 1 - P[F_{2pL, 2(n-p)L} > \frac{1}{1 + \lambda_p / \sigma^2}] \]

\[ \lambda_p = ev_{min}(HR_{uu}H^H) \]
\[ \lambda_p / \sigma^2: \text{Effective SNR before compression} \]

- (b) After compression:

\[ P_{ss} \geq 1 - P[F_{2pL, 2(m-p)L} > \frac{1}{1 + \lambda'_p / \sigma^2}] \]

\[ \lambda'_p = ev_{min}(GR_{uu}G^H) \]
\[ \lambda'_p / \sigma^2: \text{Effective SNR after compression} \]
Sensor Array Processing: Dense, Gaussian, and Co-prime

- **Dense array**

- **Gaussian compression**

- **Co-prime compression** [Pal and Vaidyanathan (2011)]

At $N = 11$ and $M = 9$, $(2M - 1)N\lambda/2 = 187\lambda/2$. 
Sensor Array Processing–Mean Case

Analytical lower bounds for the probability of subspace swap.

MSE and MSE bounds; Interfering source at $\theta_2 = \pi/188$; Average over 200 trials.
Analytical lower bounds for the probability of subspace swap.

MSE and MSE bounds; Interfering source at $\theta_2 = \pi/188$; 200 snapshots; Averaged over 500 trials.
References on Breakdown Thresholds


Intermediate Recap

Compression, whether by linear maps (eg, Gaussian or Bernoulli) or by subsampling (eg, co-prime) has performance consequences.

- The CR bound increases and the onset of threshold SNR increases. These increases may be quantified to determine where compressive sampling is viable.
Model Mismatch
From Over-determined to Under-determined

\[ y = \sum_{i=1}^{K} \psi(\nu_i)\theta_i \]

\[ y \approx [\psi(\omega_1), \cdots, \psi(\omega_n)] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \]
Model Mismatch

**Mathematical (CS) model:**

\[ s = \Psi_0 x \]

The basis (or frame) \( \Psi_0 \) is assumed, typically a gridded imaging matrix (e.g., \( n \) point DFT matrix or identity matrix), and \( x \) is presumed to be \( k \)-sparse.

**Physical (true) model:**

\[ s = \Psi_1 \theta \]

The basis \( \Psi_1 \) is unknown, and is determined by a point spread function, a Green’s function, or an impulse response, and \( \theta \) is a \( k \)-sparse and unknown.

**Key transformation:**

\[ x = \Psi \theta = \Psi_0^{-1} \Psi_1 \theta \]

\( x \) is sparse in the unknown \( \Psi \) basis, not in the identity basis.
Model Mismatch: From Sparse to Incompressible

DFT Grid Mismatch:

\[ \Psi = \Psi_0^{-1} \Psi_1 = \begin{bmatrix} L(\Delta \theta_0 - 0) & L(\Delta \theta_1 - \frac{2\pi(n-1)}{n}) & \cdots & L(\Delta \theta_{n-1} - \frac{2\pi}{n}) \\ L(\Delta \theta_0 - \frac{2\pi}{n}) & L(\Delta \theta_1 - 0) & \cdots & L(\Delta \theta_{n-1} - \frac{2\pi}{n}) \\ \vdots & \vdots & \ddots & \vdots \\ L(\Delta \theta_0 - \frac{2\pi(n-1)}{n}) & L(\Delta \theta_1 - \frac{2\pi(n-2)}{n}) & \cdots & L(\Delta \theta_{n-1} - 0) \end{bmatrix} \]

where \( L(\theta) \) is the Dirichlet kernel:

\[ L(\theta) = \frac{1}{n} \sum_{\ell=0}^{n-1} e^{i\ell \theta} = \frac{1}{n} e^{i\theta (n-1)/2} \frac{\sin(\theta n/2)}{\sin(\theta/2)}. \]

Slow decay of the Dirichlet kernel means that the presumably sparse vector \( x = \Psi \theta \) is in fact incompressible.
Sensitivity to Model Mismatch

**Question**: What is the consequence of assuming that $\mathbf{x}$ is $k$-sparse in $\mathbf{I}$, when in fact it is only $k$-sparse in an unknown basis $\Psi$, which is determined by the mismatch between $\Psi_0$ and $\Psi_1$?

**CS Inverter**: Basis pursuit solution satisfies

\[
\begin{align*}
\text{Noise-free:} & \quad \|\mathbf{x}^* - \mathbf{x}\|_1 & \leq C_0 \|\mathbf{x} - \mathbf{x}_k\|_1 \\
\text{Noisy:} & \quad \|\mathbf{x}^* - \mathbf{x}\|_2 & \leq C_0 k^{-1/2} \|\mathbf{x} - \mathbf{x}_k\|_1 + C_1 \epsilon
\end{align*}
\]

where $\mathbf{x}_k$ is the best $k$-term approximation to $\mathbf{x}$.

**Key**: Analyze the sensitivity of $\|\mathbf{x} - \mathbf{x}_k\|_1$ to basis mismatch.
Sensitivity to Model Mismatch

**Theorem:** [Chi, Scharf, P., Calderbank (TSP 2011)] Let 
\( \Psi = \Psi_0^{-1} \Psi_1 = I + E \), where \( x = \Psi \theta \). Let \( 1 \leq p, q \leq \infty \) and 
\( 1/p + 1/q = 1 \).

- If the rows \( e_{\ell}^T \in \mathbb{C}^{1 \times n} \) of \( E \) are bounded as \( \| e_{\ell} \|_p \leq \beta \), then

\[
\| x - x_k \|_1 \leq \| \theta - \theta_k \|_1 + (n - k)\beta \| \theta \|_q.
\]

The bound is achieved when the entries of \( E \) satisfy

\[
e_{mn} = \pm \beta \cdot e^{j(\arg(\theta_m) - \arg(\theta_n))} \cdot (|\theta_n|/\| \theta \|_q)^{q/p}.
\]

**Message:** In the presence of basis mismatch, exact or near-exact sparse recovery cannot be guaranteed. Recovery may suffer large errors.
Mismatch in Modal Analysis

![Graphs showing actual modes, conventional FFT, compressed sensing, and linear prediction](image)

- **Actual modes**
- **Conventional FFT**
- **Compressed sensing**
- **Linear Prediction**

**Frequency mismatch**
Mismatch in Modal Analysis

- Actual modes
- Conventional FFT
- Compressed sensing
- Linear Prediction

Damping mismatch
Mismatch in Modal Analysis

- Actual modes
- Conventional FFT
- Compressed sensing
- Linear Prediction with Rank Reduction

Frequency mismatch–noisy measurements
Noise Limited, Quantization Limited, or Null Space Limited

\[ l_1 \text{ inversions for } L = 2, 4, 6, 8 \]

\[ f_1 = 0.5 \text{ Hz}, \ f_2 = 0.52 \text{ Hz}, \ m = 25 \text{ samples}, \text{ complex Gaussian noise of variance } \sigma^2 \]. DFT frame \( \Psi \in \mathbb{C}^{n \times mL} \) with half-cell width \( (1/2nL) \).
Noise Limited, Quantization Limited, or Null Space Limited

(a) OMP for $L = 2, 4, 6, 8$

(b) OMP for $L = 8, 12, 14$
Misfocus in Optical Imaging
Misfocus in Optical Imaging

CS mask $M=1000$, $z=0.1$DOF

CS mask $M=1000$, $z=0.5$DOF

CS mask $M=1000$, $z=1$DOF

CS mask $M=1000$, $z=2$DOF
References on Model Mismatch in CS


Compressed Sensing Off The Grid
Atomic Norm Decomposition

Model:

\[ y = \sum_{i=1}^{k} \psi_k \theta_k; \quad \{\psi_k\} : \text{Atoms} \]

Atomic norm [Chandrasekaran, Recht, Parrilo, and Willsky (Allerton 2010)]:

\[ \|y\|_A = \inf_{(\theta, \psi)} \sum_{i=1}^{k} |\theta_k| \]

Atomic norm decomposition:

\[ \min \|\eta\|_A \quad \text{s.t.} \quad P_\Omega(y) = P_\Omega(\eta) \]

Atomic set:

\[ A = \left\{ \begin{pmatrix} e^{i\phi} \\ e^{i(2\pi \nu + \phi)} \\ \vdots \\ e^{i(n-1)2\pi \nu + \phi} \end{pmatrix} : \nu \in [0, 1), \phi \in [0, 2\pi) \right\} \]
Line spectra resolution:

- **Theorem:** [Candes and Fernandez-Granda 2012] A line spectrum with minimum frequency separation $\Delta_f > 4/k$ can be recovered from the first $2k$ Fourier coefficients via atomic norm minimization.

- **Theorem:** [Tang, Bhaskar, Shah, and Recht 2012] A line spectrum with minimum frequency separation $\Delta_f > 4/n$ can be recovered from most subsets of the first $n$ Fourier coefficients of size at least $m = O(k \log(k) \log(n))$.

- **Theorem:** [Chi and Chen 2013] A 2D line spectrum with minimum frequency separation $\Delta_f > 4/\left(\sqrt{n_1 n_2}\right)$ can be recovered from most subsets of the first $n$ Fourier coefficients of size at least $m = O(k \log(k) \log(n))$. 
Refernces on Off-Grid CS


Compression, whether by linear maps (eg, Gaussian or Bernoulli) or by subsampling (eg, co-prime) has performance consequences. The CR bound increases and the onset of breakdown threshold increases.