Quantitative MRI using Model-based CS

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CSA 2015 : Compressed Sensing and its Applications

Outline of Talk

- PART I
  - A General model-based CS Framework
  - Practical model-based recovery algorithm
- PART II
  - Overview of Quantitative MRI & Magnetic Resonance Fingerprinting (MRF)
  - A Compressed Sensing version of MRF

Model based CS

Basics of Compressed sensing

Signal Model:

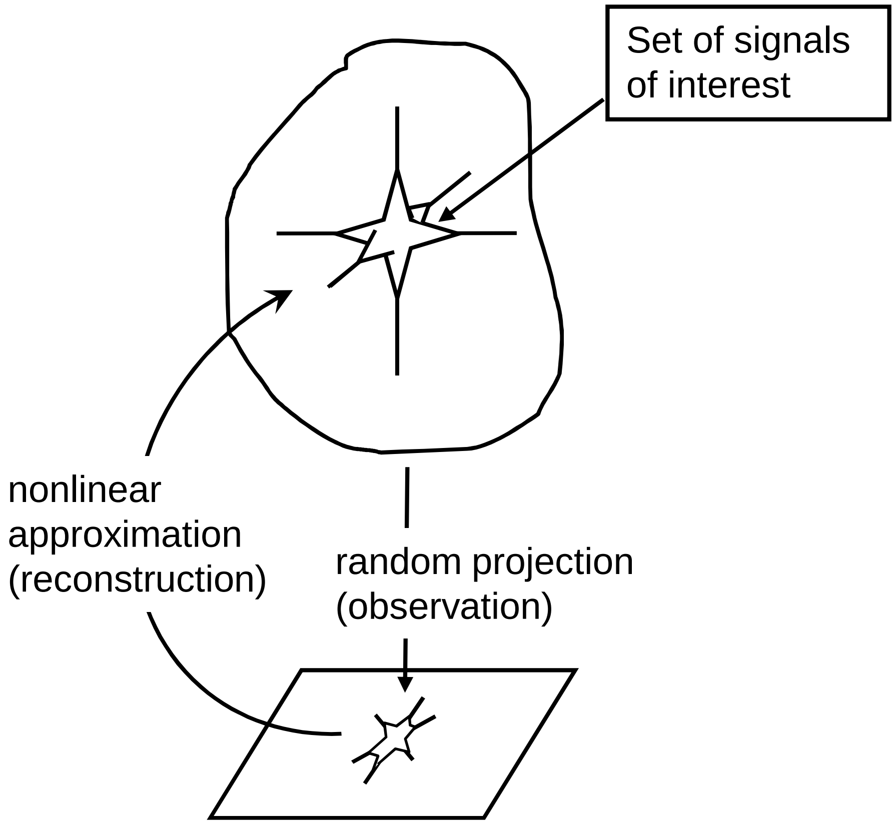
Compressed Sensing typically assumes a signal that is approximately k-sparse.

Encoder:

Use an encoder usually in the form of a random projection with e.g. RIP

Decoder:

Signal reconstruction is achieved by a nonlinear reconstruction to invert the linear projection operator on the signal set, e.g. L1, OMP, IHT, CoSaMP, AMP, etc...

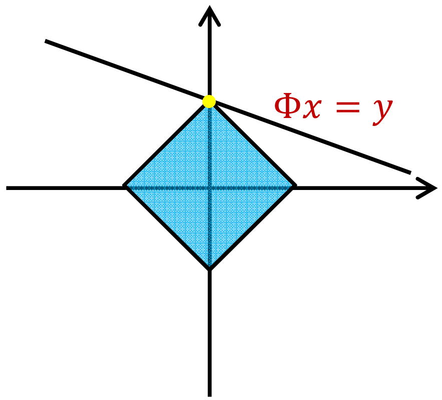


Reconstruction Algorithms

RIP enables us to replace l_0 minimization with practical algorithms, e.g.:

Relaxation: replace l_0 with l_1:

$$\hat{x} = \underset{x}{\operatorname{argmin}} \ \|x\|_1 \text{ subject to } \Phi x = y$$



Theorem [Candes 2008]:

RIP $\delta_{2k} \leq \sqrt{2} - 1 \implies$ guaranteed sparse recovery

Iterative Hard Thresholding (IHT): greedy gradient projection

$$x^{\{t+1\}} = P_{\Sigma_k}\left(x^{\{t\}} + \mu \Phi^T \left(y - \Phi x^{\{t\}}\right)\right)$$

Theorem [Blumensath, D. 2010]:

RIP $\delta_{2k} \leq 1/5 \implies$ guaranteed sparse recovery

Compressed sensing for general signal models

Signal Model:

Replace k-sparse signal model with a general signal model, e.g low rank models, union of subspace, low dimensional manifolds, …

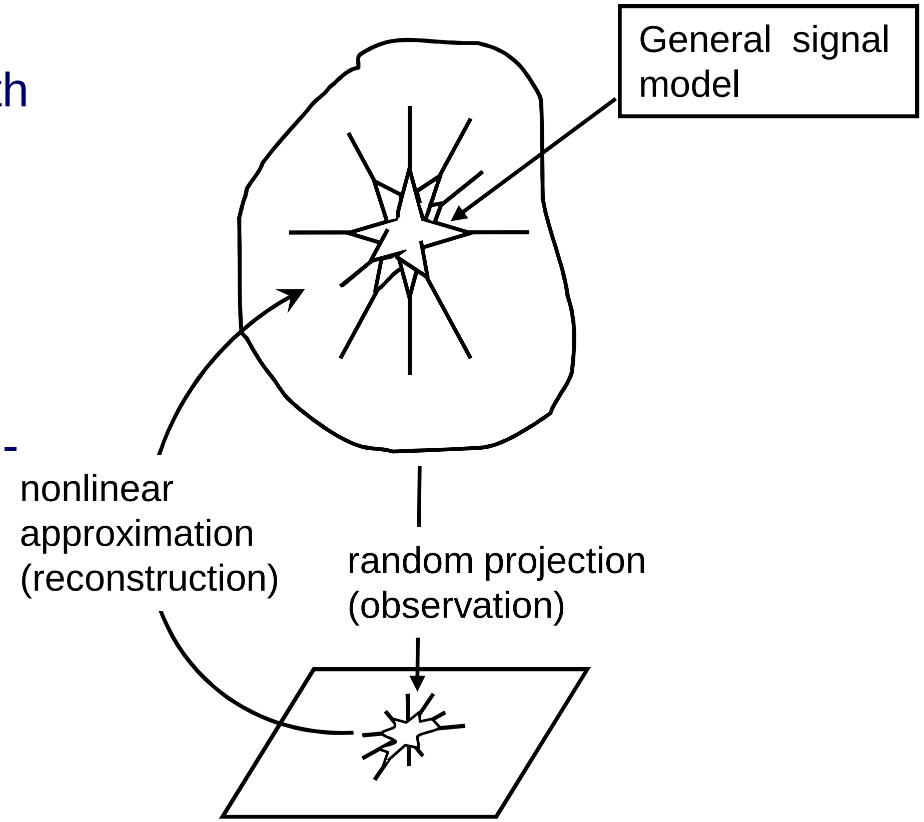
Encoder:

Information preserving, e.g. Model-based RIP

Decoder:

Atomic norm minimization?

Model-based greedy methods?



Model based CS set up

- Measurement matrix: $\Phi \in \mathbb{R}^{m \times n}$
- a general (low dimensional) signal model: $\Sigma \in \mathbb{R}^n$
- Assume a model based $(\Sigma - \Sigma)$ – RIP

$$\alpha \|z\|_2^2 \le \|\Phi z\|_2^2 \le \beta \|z\|_2^2, \qquad \forall z \in \Sigma - \Sigma$$

(can be satisfied with number of measurements: $m \sim \dim(\Sigma)$)

- We now want a practical decoder...

A Practical model-based CS Algorithm

IHT generalizes to a good (Instance Optimal) decoder for an arbitrary signal model Σ given an appropriate RIP [Blumensath 2011] :

$$x^{\{n+1\}} = P_{\Sigma}\left(x^{\{n\}} + \mu\Phi^{T}\left(y - \Phi x^{\{n\}}\right)\right)$$

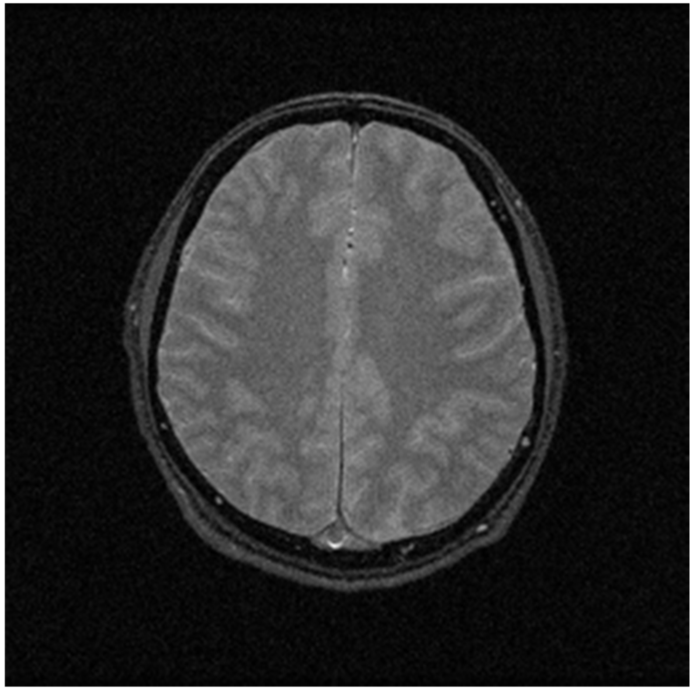
where $P_{\Sigma}(x)$ is the orthogonal projection onto Σ

Practical only if $P_{\Sigma}(x)$ can be implemented efficiently

(in practice use adaptive stepsize)

Compressive Quantitative MRI

Structural MRI is Qualitative



Standard MR images are not quantitative…

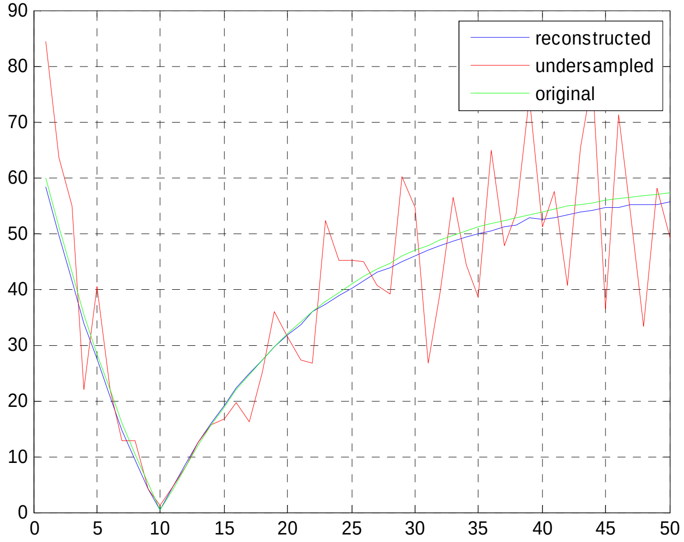
Like your digital camera [Tofts] they produce pretty pictures..

…But the process is quantitative and described by the Bloch equations (physical model):

$$\frac{\partial\, \mathbf{m}(t)}{\partial\, t} = \mathbf{m}(t) \times \gamma\, \mathbf{B}(t) - \begin{pmatrix} m^x(t)/T2 \\ m^y(t)/T2 \\ (m^z(t) - m_{eq})/T1 \end{pmatrix}$$

Quantitative MRI

- Quantitative MRI, e.g. estimation proton density, T1, T2, etc.,
- Offers better physiological information and material discrimination etc.
- Traditional approach: acquire multiple scans and estimate the exponential relaxation from multiple data points…



T1 relaxation curve fitted for each voxel

- Alternative new approach for full quantitative MRI:

  "Magnetic Resonance Fingerprinting" [Ma et al, Nature, 2013]

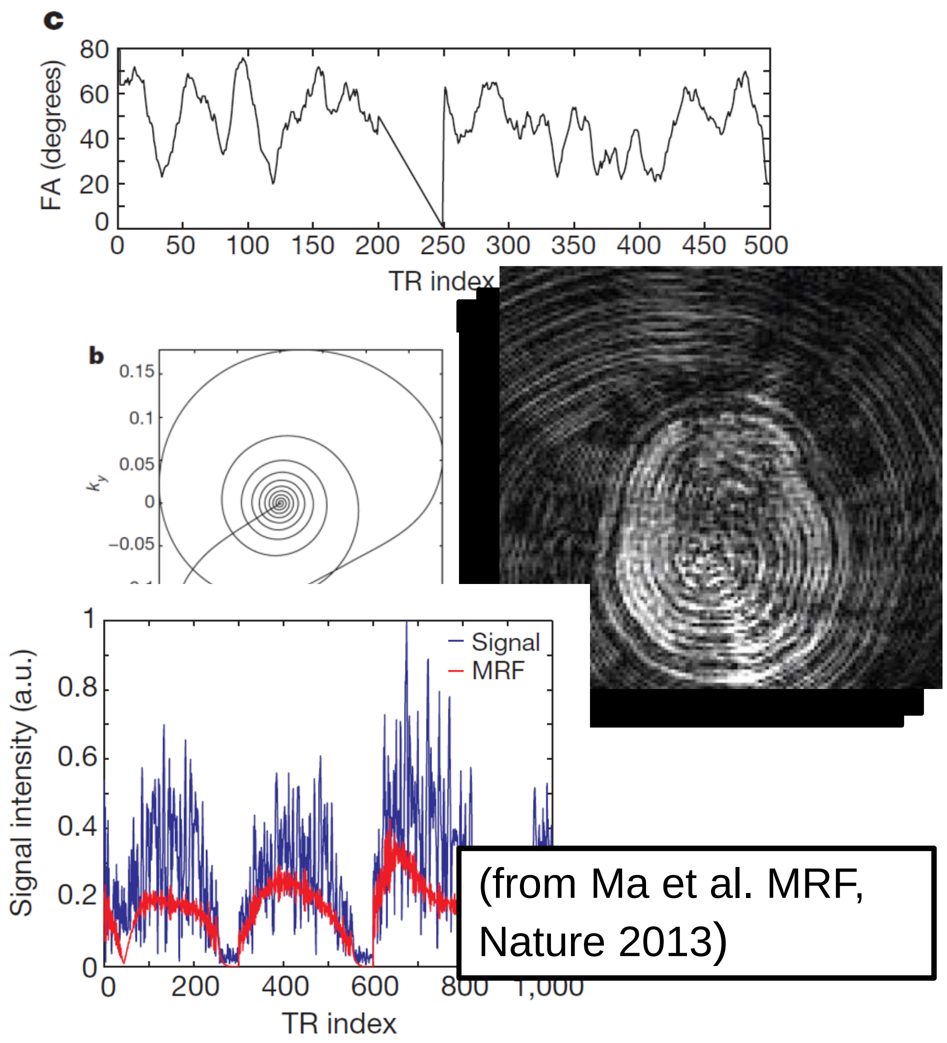
Magnetic Resonance Fingerprinting

MRF aim: simultaneous acquisition of <u>all</u> MR parameters at once!

1. Excite magnetic spin in tissue with a sequence of random RF pulses

2. Acquire image sequence from very undersampled in k-space (spiral trajectory) and back project.

3. Use dictionary, D, of predicted responses for different parameter values (fingerprints) is matched each voxel sequence



(from Ma et al. MRF, Nature 2013)

Is MRF Compressed Sensing?

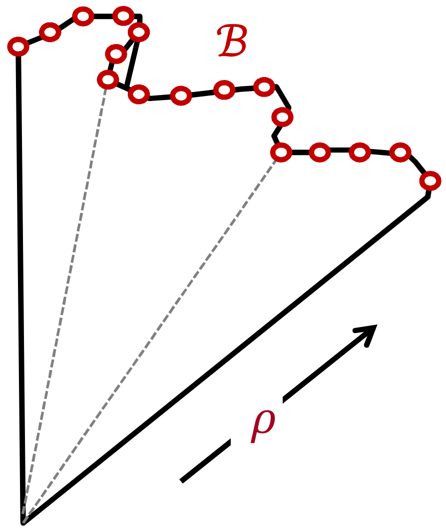
… not quite:

- Fingerprints average aliasing $\neq$ Alias cancellation (c.f. filtered Back Projection vs Iterative recon)
- Spiral k-space sampling does not provide suitable data embedding

Voxel-wise Bloch response model

We can think of the MRF dictionary D (Fingerprints) as a discretization of the Bloch magnetization response to **B**(t) with respect to the parameters T1, T2…

$$\frac{\partial \mathbf{m}(t)}{\partial t} = \mathbf{m}(t) \times \gamma \mathbf{B}(t) - \begin{pmatrix} m^x(t)/T2 \\ m^y(t)/T2 \\ (m^z(t) - m_{eq})/T1 \end{pmatrix}$$



- This essentially samples a manifold $\mathcal{B} \in \mathbb{C}^L$ for L excitation pulses

- The proton density simply scales the response defining a cone $\mathbb{R}_+\mathcal{B}$
- Full image sequence model is the N-product of this cone (N voxels):

$$X \in (\mathbb{R}_+\mathcal{B})^N \subset \mathbb{C}^{N \times L}$$

Model Projection

- MRF reconstruction is "matched filter" with voxel sequence $X_{\{i,:\}}$:

$$\hat{k}_i = \underset{k}{\operatorname{argmax}} \frac{\left|\left\langle D_k, X_{\{i,:\}} \right\rangle\right|}{\|D_k\|_2}$$

- $\mathrm{T1_i}$ and $\mathrm{T2_i}$ can be found using look up table.
- Proton density estimated as the magnitude of the correlation:

$$\hat{\rho}_i = \frac{\left\langle D_k, X_{\{i,:\}} \right\rangle}{\|D_k\|_2}$$

<u>Our interpretation</u>: this is an approximate projection onto $\mathbb{R}_+\mathcal{B}$

Excitation Scheme

- Random excitation sequences map parameter space into higher dimensional response space
- not directly part of "compressed sensing"… but still involves data embedding.
- However in order to get a RIP we require some form of *persistence of excitation* to continuously acquire new information.

Persistence of Excitation

We measure persistence of excitation through the following definition:

Definition: flatness. Let U be a collection of vectors $\{u\} \in \mathbb{C}^L$. We denote the flatness of U , $\lambda(U)$ as:

$$\lambda(U) \coloneqq \max_{u \in U} \frac{\|u\|_\infty}{\|u\|_2}$$

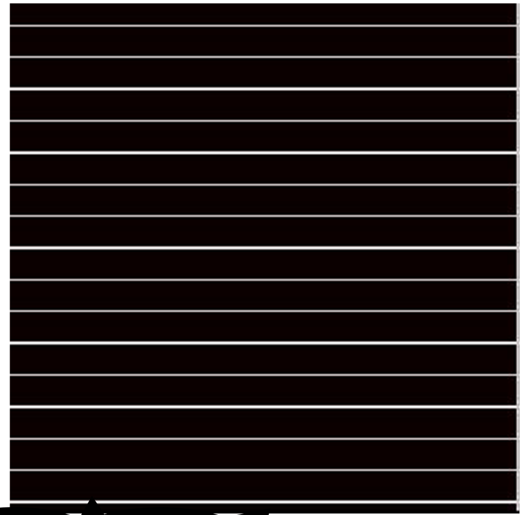
from standard norm inequalities $\mathrm{L}^{-1/2} \leq \lambda(U) \leq 1$

We assume that random pulses give us chords of $\mathbb{R}_+\mathcal{B}$, $u \in \mathbb{R}_+\mathcal{B} - \mathbb{R}_+\mathcal{B}$ are sufficiently flat (empirically true)

(similar ideas in other areas of CS)

Subsampling & model-based RIP

Signal model has no spatial structure.
Hence need to fully cover k-space



Proposed k-space sampling:

Randomized Echo Planar Imaging (EPI): uniformly subsample multiple lines in k-space with random shift

We would prefer to have $L \sim p$

Theorem [D., Puy, Vandergh ... dom EPI

If excitation is "sufficiently persistent" then random EPI with factor p undersampling achieves RIP on voxel-wise model, $(\mathbb{R}_+\mathcal{B})^N$ with a sequence length: $L \sim \mathcal{O}(\delta^{-2}p^2 \dim(\mathbb{R}_+\mathcal{B}) \log(\frac{N}{\delta}))$

Bloch response recovery via Iterated Projection (BLIP)

- Incorporate Bloch dictionary into projected gradient algorithm:
  - (1) **Gradient** : calculate for each acquisition time, t:

$$X_{:,l}^{\{n+1/2\}} = X_{:,l}^{\{n\}} + \mu F^H P(l)^T \left(P(l) F X_{:,l}^{\{n\}} - Y_{:,l} \right)$$

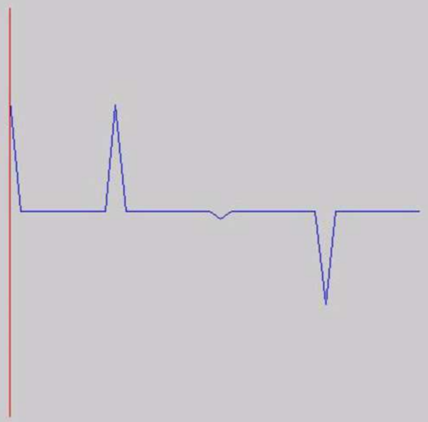
  - (2) **Projection**: for each voxel i find the atom in D most correlated to voxel sequence $X_{i,:}^{\{n+1/2\}}$ then scale and replace.

    ($\sim\mathcal{O}(L \log|\{D_k\}|)$ using a fast nearest neighbour search)

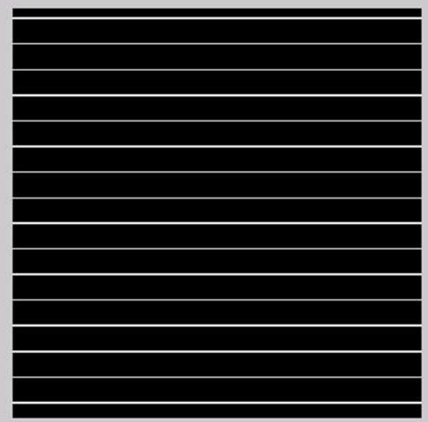
- Finally use look up table to estimate $\rho, \mathrm{T}_1, \mathrm{T}_2$

Back Projected Image Sequence

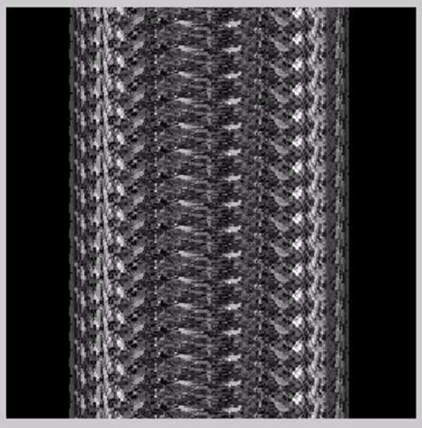
Proposed acquisition system: $Y_{:,l} = P(l)FX_{:,l}$



Random RF pulses;



Random EPI;



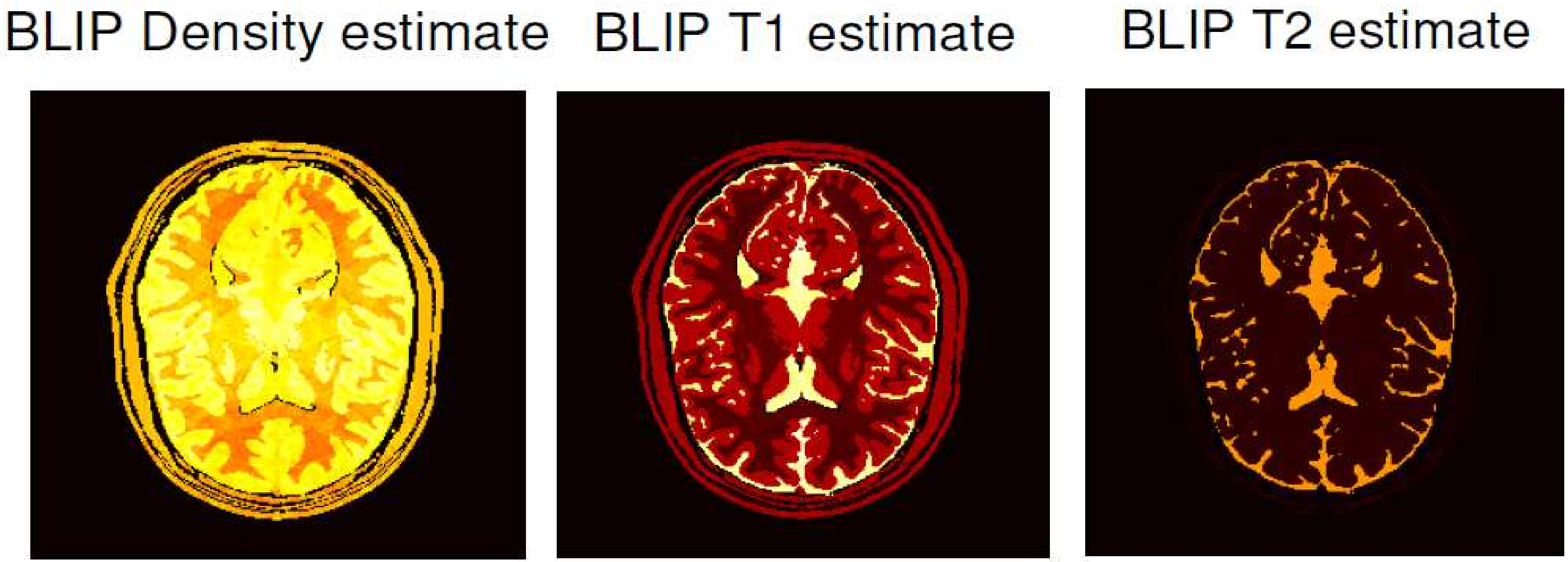
highly aliased images

Each image in the sequence is heavily aliased,… but encodes different spatial parameter information…

Together the image sequence can be restored with BLIP…

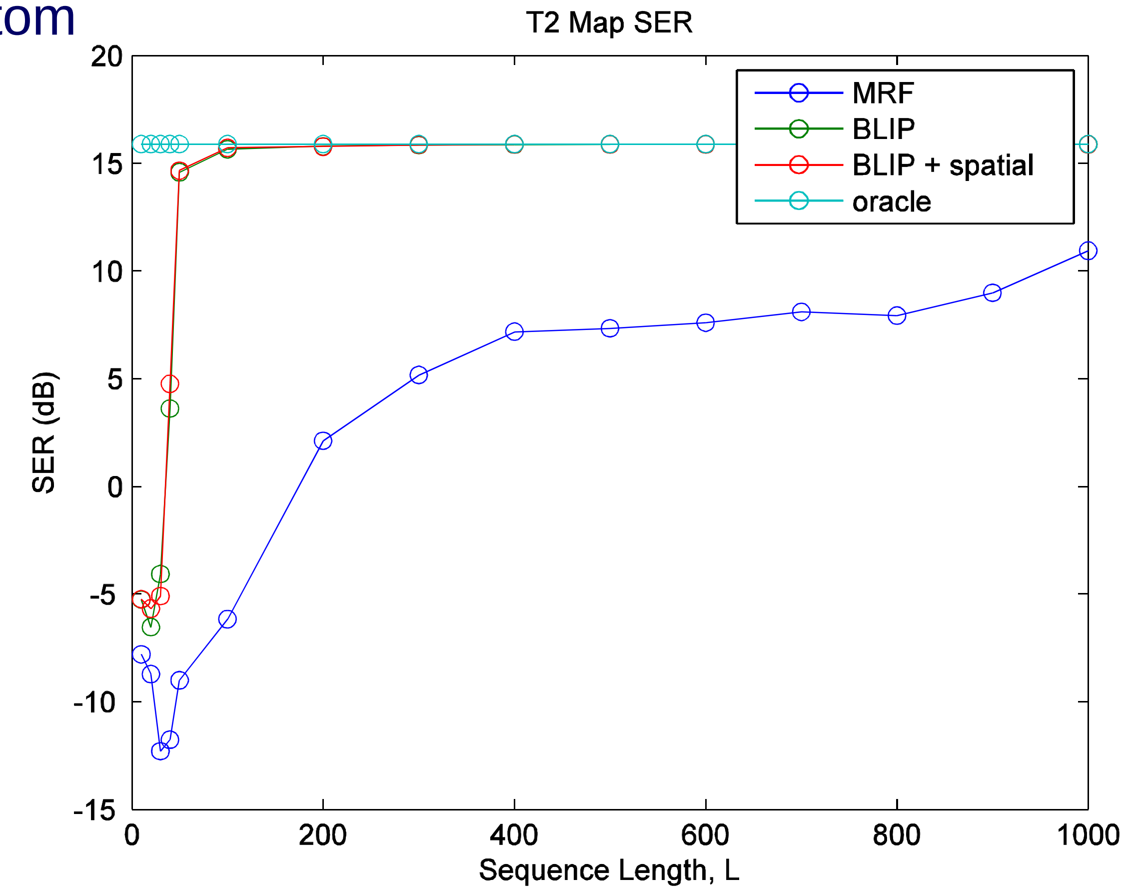
Proton density, T1 and T2

Simulation Set up: Sequence length 200; random EPI sampling at 6.25% Nyq. uniform TR and i.i.d. random flip angles applied to MNI anatomical brain phantom



Performance vs Sequence Length

Random EPI sampling at 6.25% Nyq. applied to the MNI anatomical brain phantom



BLIP gives near perfect recovery from very short pulse sequences

– significant improvement over the MRF matched filter reconstruction

Conclusions

- Model based CS gives us a new tool for CS
- Initial go at applying it to fully quantitative MR Imaging
- Developed a practical algorithm based on gradient projection onto the Bloch equations model and Random EPI sampling (BLIP)

Next…

- We need to put it on the scanner. (in progress..)
- Deduce better excitation sequences & sampling patterns
- Evaluate model inaccuracies
- Determine how best to incorporate spatial regularization

Questions