Operator equalities as means for the study of singular integral operators with Carleman shift

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In the article [1,2] we obtained a direct relation between singular integral operators A with a model involution and matrix characteristic singular integral operators: for an orientation-preserving shift it is a similarity transform $\mathcal{F}A\mathcal{F}^{-1}$ and for an orientation-reversing shift it is a transform by two invertible operators $\mathcal{H}A\mathcal{E}$. We will refer to the formulas as operator equalities.

Different applications of operator equalities to singular integral operators and to boundary value problems are considered.

In particular, in the space $L_2(\Gamma)$ we study a structure of the kernel of singular integral operators with involution

$$A_\Gamma = a_\Gamma I_\Gamma + c_\Gamma S_\Gamma + b_\Gamma W_\Gamma + d_\Gamma S_\Gamma W_\Gamma,$$

where Γ is the unit circle $\mathbb{T}$ o the real axis $\mathbb{R}$, coefficients are bounded measurable functions on Γ; $(W_\Gamma\varphi)(t) = \varphi(-t)$, $(I_\Gamma\varphi)(t) = \varphi(t)$, S_Γ is the Cauchy singular integral operator.

[1] A. A. Karelin, On a relation between singular integral operators with a Carleman linear-fractional shift and matrix characteristic operators without s hift, Boletin Soc. Mat. Mexicana Vol. 7 No. 12 (2001), pp. 235–246.

[2] A. Karelin, Aplications of operator equalities to singular integral operators and to Riemann boundary value problems, Math. Nachr. Vol. 280 No. 9-10 (2007), pp. 1108–1117.

The talk is based on a joint work with A. Tarasenko and G. Perez Lechuga