Minimal and maximal invariant Banach spaces of holomorphic functions on bounded symmetric domains

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Let D be a *Cartan domain* of rank r and genus p in $\mathbb{C}^d$. The group G of all holomorphic automorphisms of D acts projectively and irreducibly on holomorphic functions on D via

$$U^{(\lambda)}(g)(f)(z) := (J(g^{-1})(z))^{\lambda/p} f(g^{-1}(z)), \quad z \in D, \quad g \in G,$$

where "J" stands for the complex Jacobian and λ is the "Wallach parameter".

We study the minimal and maximal Banach spaces of holomorphic functions on D which are isometrically invariant under $U^{(\lambda)}(G)$. In particular - we prove their existence and uniqueness, establish their duality with respect to the (unique) $U^{(\lambda)}(G)$ - invariant inner product, and identify them concretely as Besov-type spaces. It follows that the maximal space $M^{(\lambda)}$ consists of all holomorphic functions F on D for which $F(z)h(z,z)^{\lambda/2}$ is bounded, where $h(z,w)$ is the "Jordan determinant" (given by $det(I - zw^*)$ in the matrix domains), and the minimal space $\mathfrak{M}^{(\lambda)}$ is the space of all holomorphic functions f on D for which $f(z)h(z,z)^{\lambda/2}$ is integrable with respect to the (unique) G-invariant measure on D.

This study is extended to points λ in the *pole set* of D in the cases where D is of tube type and the highest quotient of the composition series of the $U^{(\lambda)}(G)$-invariant spaces is unitarizable (i.e. $s := d/r - \lambda$ is a positive integer). The main tool in this study is the *intertwining formula*

$$N^s(\frac{\partial}{\partial z})U^{(\lambda)}(g) = U^{(p-\lambda)}(g)\, N^s(\frac{\partial}{\partial z}), \quad \forall g \in G,$$

where $N(z)$ is the *determinant* polynomial of the Euclidean Jordan algebra associated with D. For instance, if $\lambda = 0$ (and thus $U^{(0)}(g)f := f \circ g^{-1}$) and

$s := d/r$ is a positive integer, then

$$M^{(0)} = \{F \;\text{ holomorphic in }\; D;\; \|F\| := \sup_{z\in D} |N^s(\frac{\partial}{\partial z})F(z)|h(z,z)^s < \infty\},$$

$$\mathfrak{M}^{(0)} = \{f \;\text{ holomorphic in }\; D;\; \|f\| := \int_D |N^{2s}(\frac{\partial}{\partial z})(N^s(z)f(z))|\; dm(z) \;<\; \infty\}.$$

These results generalize the well-known facts concerning the maximality of the Bloch space and the minimality of the Besov space $B^1_{1,1}$ among Möbius-invariant Banach spaces of holomorphic functions on the unit disk.