Bounded quasi-selfadjoint operators, their Weyl functions, and special block operator Jacobi matrices

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A bounded operator T in a separable Hilbert space $\mathfrak{H}$ is called quasi-selfadjoint if $\ker(T - T^*) \neq \{0\}$ and $\mathfrak{N}$-quasi-selfadjoint if $\mathfrak{N} \supseteq \operatorname{ran}(T - T^*)$, where $\mathfrak{N}$ is a subspace of $\mathfrak{H}$. An $\mathfrak{N}$-quasi-selfadjoint operator T is called $\mathfrak{N}$-simple if the linear hull of $\{T^n\mathfrak{N},\ n = 0, 1, \ldots\}$ is dense in $\mathfrak{H}$. We study the $\mathfrak{N}$-Weyl function $M(z) = P_{\mathfrak{N}}(T - zI_{\mathfrak{H}})^{-1} \upharpoonright \mathfrak{N}$ of an $\mathfrak{N}$-quasi-selfadjoint operator and define Schur transformation and Schur parameters of $M(z)$. The main result is that any $\mathfrak{N}$-quasi-selfadjoint and $\mathfrak{N}$-simple operator is unitarily equivalent to an operator given by a special block operator Jacobi matrix constructed by means of the Schur parameters of its $\mathfrak{N}$-Weyl function.

The talk is based on a joint work with L. Klotz.