

# An index formula for classes of pseudodifferential operators on non-compact spaces

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The Atiyah-Singer index formula gives a topological formulation for the Fredholm index of elliptic pseudodifferential operators on compact spaces, generalizing the fundamental Gohberg-Krein index theorem for operators on the circle. On non-compact spaces, many difficulties arise: for a start, ellipticity does not suffice, in general, to ensure Fredholmness. In this talk, we consider a class of pseudodifferential operators on  $\mathbb{R}^n$ , or more generally, on a Lie manifold, that extend at infinity to a suitable compactification, in that they are generated by a given class of vector fields that are tangent to the boundary of the compactification. As particular cases, one obtains several well-known pseudodifferential calculi on  $\mathbb{R}^n$ . In case the vector fields actually vanish at infinity, we give Fredholm conditions depending on ellipticity and invertibility of a complete symbol on spheres at the boundary. We then extend Atiyah-Singer's index formula to this class; to this effect, we use homotopy to asymptotically constant symbols. This result can be applied to perturbed Dirac operators  $\mathcal{D} + V$ , where  $V$  is an unbounded potential.

The talk is based on joint work with V. Nistor.