

Essential Spectrum of Multivariable Matrix-Valued Toeplitz Operators

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Classical Toeplitz operators were defined on the Hardy space for the unit disk with a major focus being on the invertibility or Fredholmness of them. For a Toeplitz operator with a continuous, matrix-valued symbol, the essential spectrum was determined more than fifty years ago by Gohberg and Krein. Since then, many researchers have considered such questions for Toeplitz operators with more general symbols and on other Hilbert spaces of holomorphic functions such as the Bergman space. More recently, generalizations to the multivariable context have been considered with new phenomena present and many different techniques involved.

In joint work with J. Sarkar, we obtain necessary conditions for the semi-Fredholmness of a Toeplitz operator with a matrix-valued holomorphic symbol on a Hilbert space of holomorphic functions on the unit ball. Examples of such spaces are the Hardy or Bergman spaces on the unit ball or the Drury-Arveson space. The techniques used include the Berezin transform in this context as well as the Taylor spectrum defined in terms of the Koszul complex. Relations of these questions to the Corona problem are also discussed.