Szegö-Widom Limit theorems for band-dominated operators with almost periodic diagonals

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The classical Szegö-Widom Limit Theorem describes the asymptotics of the determinants of block Toeplitz matrices

$$T_{nN}(A) = (A_{j-k})\,, \quad 0 \le j,k \le n-1, \quad A_k \in \mathbb{C}^{N\times N}$$

as n goes to infinity. An equivalent way of describing these block Toeplitz matrices is to say that all their diagonals are periodic sequences with period N and that the matrix size is a multiple of N.

The goal of the work presented is to generalize the limit theorem to matrices whose diagonals are almost periodic sequences. The most prominent example arises from the finite sections of the almost Mathieu operator,

$$M = U_1 + aI + U_{-1}$$

where $U_{\pm 1}$ are the forward and backward shift on ℓ^2 and a stands for the sequence $a(n) = \alpha + \beta \sin(2\pi\xi n + \delta)$, determining the entries of the main diagonal. As suggested by the block case, the size of the finite sections has to be chosen in a suitable way to yield reasonable results.

A somewhat surprising feature is that the asymptotics may be significantly different. For the explanation of this fact, one has to resort to some deep results in diophantine approximation theory. For example, in the case of the almost Mathieu operator the transcendence properties of ξ play a crucial role.

As all classical methods of proving the Szegö-Widom Theorem fail to generalize, a new algebraic approach is used to compute the determinants asymptotically.

This talk is based on joint work with S. Roch and B. Silbermann.