Inverse scattering on the line for Schrödinger operators with singular Miura potentials

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We study direct and inverse scattering problems for one-dimensional Schrödinger operators with highly singular Miura potentials $q \in H^{-1}(\mathbb{R})$, i.e., potentials of the form $q = u' + u^2$ for some $u \in L_2(\mathbb{R})$. Under some additional assumptions there exist unique Riccati representatives u_+ and u_- that are integrable respectively at $+\infty$ and $-\infty$, and there is a well-defined reflection coefficient r that determines u_+ and u_- uniquely. We show that the map $(u_+, u_-) \mapsto r$ is continuous with continuous inverse and obtain an explicit reconstruction formula. Among potentials included are, e.g., potentials of Marchenko–Faddeev class and their perturbations by compactly supported distributions from $H^{-1}(\mathbb{R})$ (e.g., delta-functions and regularized Coulomb $1/x$-type interactions) and some highly oscillating unbounded potentials.

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