A theorem of Heinz Langer on the factorization of selfadjoint operator polynomials

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In 1973 Langer [1] proved the following theorem.

Theorem. Let $L(\lambda)$ be a selfadjoint operator polynomial of degree n and $[a, b] \subset \mathbb{R}$. If

$$L(a) \ll 0, L(b) \gg 0, L'(\lambda) \gg 0 (a \leq \lambda \leq b),$$

then $L(\lambda)$ admits a factorization

$$L(\lambda) = M(\lambda)(\lambda I - Z),$$

where $M(\lambda)$ is an operator polynomial of degree $n - 1$, and Z is similar to a selfadjoint operator. Moreover, $M(\lambda)$ is invertible for all $\lambda \in [a, b]$ and $\sigma(Z) \subset [a, b]$.

I will try to explain the main ideas of the proof of this remarkable theorem, and its influence on the development of the spectral theory of selfadjoint polynomials and analytic operator functions.

[1] H. Langer, *Über eine Klasse nichtlinearer Eigenwertprobleme.* Acta Sci. Math. (Szeged), **35**(1973), 73-86.