On the location of roots of Hermite-Biehler polynomials

M. Tyaglov

The well-known Hermite-Biehler theorem claims that a univariate monic polynomial s of degree k has all roots in the open upper half-plane if and only if $s = p + iq$ where p and q are real polynomials of degree k and $k-1$ resp. with all real, simple and interlacing roots, and q has a negative leading coefficient. Considering roots of p as cyclically ordered on $\mathbb{R}P^1$ we show that the open disk D in $\mathbb{C}P^1$ having a pair of consecutive roots of p as its diameter is the maximal univalent disk for the function $R = \frac{q}{p}$. In particular, each disk D contains at most one root of the polynomial s. This solves a special case of the so-called Hermite-Biehler problem.

The talk is based on a joint work with B. Shapiro and V. Kostov.