

Hamiltonians with Riesz bases of eigenvectors and Riccati equations

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We consider the algebraic Riccati equation

$$A^*X + XA + XQ_1X - Q_2 = 0,$$

which appears in the problem of optimal control of a linear system, for the case that A is a normal operator with compact resolvent and Q_1, Q_2 are unbounded, selfadjoint and nonnegative. Using the well-known correspondence of solutions X to invariant graph subspaces of the Hamiltonian

$$T = \begin{pmatrix} A & Q_1 \\ Q_2 & -A^* \end{pmatrix},$$

we prove the existence of infinitely many selfadjoint solutions. Our main tools are a Riesz basis with parentheses of generalised eigenvectors of T and two indefinite inner products associated with T . We also obtain conditions which yield nonnegative, nonpositive, and bounded solutions.