Singular perturbation approximation of bilinear systems with applications to stochastic control

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We study balanced model reduction of stable bilinear control systems. In doing so we adopt ideas from balancing of linear systems and decompose the equations into a states that are well controllable and observable and states that are hardly so (cf. [1, 2]). The decomposition admits a splitting of the dynamics into fast and a slow subspaces with the fast subspace being invariant and asymptotically stable when the correponding Hankel singular values are sufficiently small. A reduced-order model is then obtained by sending the small Hankel singular values to zero [3].
Our approach resembles the well-known averaging method for systems with slow and fast degrees of freedom [4], and we give an asymptotic error bound that holds on finite time intervals and which depends upon the small Hankel singular values and the decay rate of the control as time goes to infinity. We illustrate the method with an example from stochastic control (density evolution of a dragged Brownian particle) and discuss issues of structure preservation and positivity.

References

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