Nonlinear Model Reduction for Rubber Components in Vehicle Engineering

S. Herkt[1], R. Pinnau[2], and K. Dreßler[3]

[1]Fraunhofer ITWM, Kaiserslautern, sabrina.herkt@itwm.fraunhofer.de
[2]Uni Kaiserslautern, rene.pinnau@uni-kl.de
[3]Fraunhofer ITWM, Kaiserslautern, klaus.dressler@itwm.fraunhofer.de

In vehicle engineering, rubber parts play an important role. Components like bushings and tires are designed to damp and suspend undesired vibrations.
For the simulation of full vehicle effects, multibody simulation methods are extensively used and well established. Here, the description of rubber components relies either on spring-damper formulations with nonlinear characteristics or on linear modal model reduction of FEM models (e.g. Craig-Bampton method [2]). Nonlinear model reduction methods have a great potential in this field but are not productively employed yet.

The reduction of Finite Element systems with material and geometrical nonlinearities starts from the equation

$$M\ddot{u} + C\dot{u} + R(u) = f$$

with constant mass and damping matrices M, C. The term of inner forces $R(u)$ contains nonlinear effects induced by geometrical and material nonlinearities.
One possibility to define a subspace for the projection of nonlinear systems is the method of *Proper Orthogonal Decomposition (POD)*. It can be regarded as an approach to approximate a set of solution data (*snapshots*) with a low dimensional subspace.
The reduction of nonlinear problems introduces additional technical difficulties as the model equations depend on the current state u:

$$\begin{aligned} M\ddot{u} + C\dot{u} + R(u) &= f_{ext} \\ P' \cdot M \cdot (\ddot{P\alpha}) + P' \cdot C \cdot (\dot{P\alpha}) + P' \cdot R(P\alpha) &= P' \cdot f_{ext} \end{aligned}$$

with $\tilde{u}(x,t) = \sum_{i=1}^{l} \alpha_i(t)\varphi_i(x) = P\alpha$ and the projection matrix $P = [\varphi_1, \ldots, \varphi_l]$.
The effort to set up the nonlinear term $R(P\alpha)$ requires the transformation of the reduced state α to the full dimensional variable $\tilde{u} = P\alpha$. In many industrial problems the nonlinearity is not explicitly known or can not be expressed in the reduced variable.
Especially in the case of collaboration with commercial software, the effort for equation setup exceeds the savings obtained from reduction of the degrees of freedom.
To decouple the reduced surrogate system from the full model, a lookup table approach is proposed (see [1]). It makes use of the training step with the full model which is necessary to set up the POD basis. The nonlinear term of inner forces and the stiffness matrix are output for each solution snapshot. This data is stored in a lookup table $\{u_i, R(u_i), K(u_i)\}$ for the reduced system.

This contribution shows the method for systems with material and geometrical nonlinearities. Furthermore, an outlook is given on the application to viscoelastic materials, which yield velocity- and time-dependent equations in the full model.

References

[1] S. Herkt, *Model Reduction of Nonlinear Problems in Structural Mechanics: Towards a Finite Element Tyre Model for Multibody Simulation*, PhD Thesis, TU Kaiserslautern, 2009

[2] R. R. Craig, Jr. *Coupling of Substructures for Dynamic Analyses: an Overview*, AIAA-2000-1573, 2000