On Dominant Poles and Model Reduction of Second Order Time-Delay Systems

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Vibrating systems appear in a wide variety of problems. There is a tendency to analyze and design systems of ever increasing size. Large scale models are more difficult to analyze. As a result, it is also harder to develop control algorithms. Model order reduction (MOR) is of great importance since it reduces the size of the model while keeping most of its characteristics.
In this talk, we study the dominant pole algorithm [3] for second order systems. The dominant poles and corresponding eigenvectors lead to a reduced system. The algorithm is based on the original system matrices and so no linearization is required. Subspace acceleration and deflation of converged dominant poles are needed to improve the convergence of the algorithm.
Sometimes, a control term is added to improve the stability of the system [1]. We consider a control with delay τ [2]

$$\left\{ \begin{array}{rcl} M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + G\dot{x}(t-\tau) + Fx(t-\tau) & = & bu(t) \\ y(t) & = & d^*x(t) \end{array} \right. \tag{1}$$

where $M, C, K \in \mathbb{R}^{n\times n}$ are mass, damping and stiffness matrices, respectively, and $G, F \in \mathbb{R}^{n\times n}$ are control matrices, $b, d, x \in \mathbb{R}^n$, x is the state space, $u(t), y(t) \in \mathbb{R}$ are input and output, respectively. System (1) is called a second order time-delay system (TDS) and its frequency response function (FRF) is

$$H(s) = d^*(s^2M + sC + K + sGe^{-s\tau} + Fe^{-s\tau})^{-1}b. \tag{2}$$

If all eigenvalues are simple then $H(s)$ can be expressed as

$$H(s) = \sum_{i=1}^{\infty} \frac{R_i}{s-\lambda_i}. \tag{3}$$

where λ_i, $i \in \mathbb{N}$ are the eigenvalues of the TDS and the R_i are the residues. Unlike the ODE case, system (1) has no finite dimension linearization and the computation of residues, R_i, in (3) relies on a more general theory. Using this result, we can adapt the dominant pole algorithm (DPA) to TDSs. We will present the theory for DPA for delay systems, develop an algorithm and show numerical results for an academic example.

References

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