Balancing based structure preserving order reduction for port-Hamiltonian systems

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Mathematical models arising from port-based network modeling have a geometric structure, which can be regarded as a generalization of the geometric formulation of analytical mechanics into its Hamiltonian form. These geometric dynamical system models have been called port-Hamiltonian systems. Port- Hamiltonian systems are compositional in the sense that any power-conserving interconnection of port-Hamiltonian systems is again port-Hamiltonian. The state space dimension of mathematical models arising from network modeling easily becomes very large, and thus there is a need for order reduction methods. However, since the interconnection structure and other properties of PH systems are useful and insightful, it is desirable to approximate high order PH systems with lower order PH systems.

In this paper an approach to reduce a minimal port-Hamiltonian (PH) system in a structure preserving manner via a balanced Kalman decomposition is studied. In [1] an approach is presented to reduce a non-minimal PH system to a minimal PH system via the Kalman decompositions for controllability and observability. This approach has been extended to the nonlinear case in [2]. Furthermore, such Kalman decomposition yields opportunities to reduce a minimal PH system to a PH system of lower order via either the controllability decomposition or the observability decomposition, which can be interpreted as reduction via an effort or a flow constraint respectively. These two possible decompositions do not result in the same reduced order PH system, i.e., one is changing the interconnection and damping structure, the other one is changing the Hamiltonian.

Here we combine the two decomposition approaches, based on the structure of balanced realizations, in which case the almost non-controllable and almost non-observable subspaces coincide. Using the Kalman decomposition based upon a splitting into two subspaces corresponding to the large and the small Hankel singular values, respectively, we study a balancing based order reduction method that preserves the PH structure of the system, and thus the passivity and stability properties of the system. Such reduction procedure however may not preserve the Hankel singular values and the balanced form, in contrast to standard balanced truncation. In this presentation we study the relation between the original balanced PH system and the reduced order system.

References

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