Balanced model reduction of gradient systems

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Joint work with
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Change of topic from abstract

Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...

Meow!
Original intention

Balancing based structure preserving order reduction for port-Hamiltonian (pH) systems

- Based on work of Polyuga and Van der Schaft for linear pH systems, to reduce non-minimal pH system to a minimal pH systems (precise reduction).
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• Based on work of Polyuga and Van der Schaft for linear pH systems, to reduce non-minimal pH system to a minimal pH systems (precise reduction).

• Extension to nonlinear case done by Scherpen and Van der Schaft.

• Based on either observability or controllability, resulting in different models.

• Combination should result in approximate model order reduction. However, unfinished, equations are still "ugly".
Outline

For gradient system we do have nice results!

Outline:

• **Introduction**
  • gradient systems
  • linear balanced realizations
• Linear gradient systems and balancing
• Nonlinear gradient systems and balancing
• Concluding remarks
Introduction

• Gradient systems large class of systems, i.e., linear and nonlinear RLC circuits, Brayton-Moser systems, etc.
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• Gradient form of drift vector field, along with pseudo-Riemannian metric.
Introduction

- Gradient systems large class of systems, i.e., linear and nonlinear RLC circuits, Brayton-Moser systems, etc.
- Popular in circuits / systems / control literature of 70’s
- Gradient form of drift vector field, along with pseudo-Riemannian metric.
- System theoretic relation with symmetric systems, i.e., input vector field and output map such that overall system is symmetric. For linear systems: $H(s) = H^T(s)$, with $H$ transfer matrix.
Introduction: gradient systems

Nonlinear gradient system:

\[ G(x) \dot{x} = -\frac{\partial P}{\partial x}(x) + \frac{\partial}{\partial x}(x)u \]
\[ y = h(x) \]

\( G(x) = G^T(x) \) invertible pseudo-Riemannian metric, \( P \) mixed potential function.
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Linear gradient system with mixed potential function \( \frac{1}{2}x^TPx \):

\[ G \dot{x} = -Px + C^Tu \]
\[ y = Cx \]

\( P = P^T, \ G = G^T \neq 0, \ H(s) = C(sI - G^{-1}P)^{-1}G^{-1}C^T = H(s)^T. \)
Introduction: gradient systems

Linear gradient system:

\[ G \dot{x} = -Px + C^T u \]
\[ y = Cx \]

**Example:** \( x \) currents/voltages through inductors/over condensators,

\[ G = \text{blockdiag}\{L, -C\}, \]

\( P \) matrix containing resistors, conductors and interconnection structure

\( u \) sources, and \( y \) corresponding currents or voltages (power outputs).
Introduction: gradient systems

Model order reduction for large scale gradient systems should "preserve" as much as possible:

- Input/output structure (interconnection structure).
- Gradient structure.

Focus presentation: combine

- balanced realization based model reduction
- preserve gradient structure.
Introduction: Linear balancing review

Continuous-time, causal linear input-output system $S : u \to y$ with impulse response $H(t)$.
If $S$ is also BIBO stable then the system **Hankel operator:**

$$
\mathcal{H} : L^m_2[0, +\infty) \to L^p_2[0, +\infty) \\
\hat{u} \to \hat{y}(t) = \int_0^\infty H(t + \tau)\hat{u}(\tau) \, d\tau.
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**Time flipping** operator $\mathcal{F} : L^m_2[0, +\infty) \rightarrow L^m_2(-\infty, 0]$
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$$
\mathcal{H}(\hat{u}) = S \circ \mathcal{F}(\hat{u})
$$
Introduction: Linear balancing (continued)

\[ \mathcal{H} = OC, \]  with the controllability and observability operators \( C \) and \( O \).

with \( \sigma_i \) are Hankel singular values, i.e., \( \sigma_i^2 \) are eigenvalues of \( \mathcal{H}^*\mathcal{H} \).
Introduction: Linear balancing *(continued)*

\[ \mathcal{H} = \mathcal{O} \mathcal{C}, \] with the **controllability** and **observability operators** \( \mathcal{C} \) and \( \mathcal{O} \).

with \( \sigma_i \) are **Hankel singular values**, i.e., \( \sigma_i^2 \) are eigenvalues of \( \mathcal{H}^* \mathcal{H} \).

\((A, B, C)\) as. stable state space realization of \( S \) of order \( n \).

- \( \sigma_i^2 \) are eigenvalues of \( M W \), where \( W \geq 0 \) and \( M \geq 0 \) are the usual **controllability** and **observability Gramians** fulfilling

\[
AW + WA^T = -BB^T
\]

\[
A^T M + MA = -C^T C
\]
Introduction: Linear balancing (continued)

\((A, B, C)\) is minimal \(\iff M > 0\) and \(W > 0\).

If \((A, B, C)\) is minimal and as. stable, then there exists a state space representation where

\[
\Sigma := M = W = \begin{pmatrix}
\sigma_1 & 0 \\
\vdots & \ddots \\
0 & \sigma_n
\end{pmatrix}
\]

\(\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0\) Hankel singular values. Then system is in balanced form.
Outline

• Introduction
  • gradient systems
  • linear balanced realizations

• **Linear gradient systems and balancing**

• Nonlinear gradient systems and balancing

• Concluding remarks
Linear gradient systems and balancing

Consider linear system \( \dot{x} = Ax + Bu, \ y = Cx \)
\( x \in \mathbb{R}^n, \ u \in \mathbb{R}^m, \ y \in \mathbb{R}^m. \)

**Gradient system** if there exists \( G = G^T \neq 0 \) satisfying

\[
A^T G = GA, \quad B^T G = C
\]

Since \( H(s) = H^T(s) \), \( G \) satisfying (*) is **unique** for controllable and observable system.

With \( P = -GA = P^T \):

\[
G \dot{x} = -Px + C^T u \quad y = Cx
\]
Linear gradient systems and balancing

The controllability and observability Gramian $W$ and $M$ of $(A, B, C)$ are unique solutions of

$$AW + WA^T = BB^T, \quad A^T M + MA = C^T C$$

Pre- and postmultiplying first eq. by $G$, and using gradient cond.

$$GAWG + GW A^T G = GB B^T G \iff$$

$$A^T (GWG) + (GWG) A = C^T C$$

implying $GWG = M$. 
Linear gradient systems and balancing

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implying $GWG = M$.

In \textbf{balanced coordinates} with distinct Hankel s.v.’s:

\[ G\Sigma G = \Sigma \Rightarrow G = diag\{\pm 1\} \quad (G > 0 \Rightarrow G = I) \]
Linear gradient systems and balanced trunc.

Remark: If some Hankel s.v.’s are equal, coordinates can be chosen such that $G = \text{diag}\{\pm 1\}$

Remark: Passivity conditions are easily obtained.
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**Proposition:** Consider gradient system with \( \sigma_1 \geq \cdots \sigma_k >> \sigma_{k+1} \geq \cdots \geq \sigma_n \) in balanced form with \( G = \text{diag}\{\pm 1\} \). Truncate \( x_{k+1}, \ldots, x_n \). Then reduced order model is gradient system

\[
\dot{\hat{G}}\hat{x} = \hat{P}\hat{x} + \hat{C}^T u, \quad \hat{x} \in \mathbb{R}^k
\]

\[
\hat{y} = \hat{C}\hat{x}
\]

\( \hat{G} = \text{diag}\{\pm 1\} = G_{11}, \hat{P} = P_{11}, \hat{C} = C_1. \)
Linear gradient systems and singular pert.

**Proposition:** Consider gradient system with
\[ \sigma_1 \geq \cdots \sigma_k \gg \sigma_{k+1} \geq \cdots \geq \sigma_n \] in balanced form with
\[ G = \text{diag}\{\pm 1\}. \] Apply \( \dot{x}_{k+1} = \cdots = \dot{x}_n = 0 \), then again reduced order gradient system with \( \hat{G} = G_{11} \), and \( \hat{P} = \hat{P}^T \)

\[ \hat{P} = P_{11} - P_{12}P_{22}^{-1}P_{21} \]

and with output equation

\[ \hat{y} = \hat{C}\hat{x} + \hat{D}u \]

where \( \hat{C} := C_1 - C_2P_{22}^{-1}P_{21} \) and \( \hat{D} := C_2P_{22}^{-1}C_2^T \).
Linear gradient systems: electrical circuits

In balanced coordinates, inductors and condensators all transformed to $\pm 1$. **Mixed potential** function normally given as

$$\frac{1}{2}i^T R i - \frac{1}{2}v^T G v + i^T \Lambda v$$

with $R$ resistors, $G$ conductors, $\Lambda$ interconnection matrix (topology of circuit), and $i$ and $v$ inductor currents and capacitor voltages.
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In balanced coordinates, structure mixed potential may be lost. After truncation, circuit realization may also require additional transformers.
Linear gradient system: the cross Gramian

Consider so-called cross Gramian $X$, with

$$WM = X^2,$$

and $X$ unique solution of **Sylvester** equation $AX +XA = BC$. In fact, $X = WG = G^{-1}M$. Thus, in balanced coordinates

$$X = \Sigma G \Rightarrow X = diag\{\pm \sigma_i\}$$
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**Corollary:** Assume $\sigma_i \neq \sigma_j$, $\forall i, j$, Let $\bar{x} = Sx$ be s.t. $SXS^{-1} = \text{diag}(\pm \sigma_1, \pm \sigma_2, \cdots, \pm \sigma_n)$. Then there exists a diagonal matrix $D$ such that $DS$ is a balancing transformation.
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Nonlinear gradient systems and balancing

Nonlinear gradient system with $G(x) = G^T(x)$ invertible:

\[ G(x) \dot{x} = -\frac{\partial P}{\partial x}(x) + \frac{\partial h}{\partial x}(x)u \]

\[ y = h(x) \]
Nonlinear gradient systems and balancing

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- Recall BM nonlinear circuits are gradient systems.
- External characterization given in Cortes et. al. (2005), entailing two different prolongations of system.
- Main idea: symmetry obtained for prolongations in observability and accessibility.
- Linear case special case, nonlinear needs prolongations, complicating balancing procedure!
Nonlinear systems: balancing

Smooth system

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{align*}
\]

where \( u \in \mathbb{R}^m \), \( y \in \mathbb{R}^p \), and \( x \in M \) (manifold of dim \( n \)).

Assumptions:

- \( f(0) = 0 \), 0 as. stable eq. point for \( u = 0, x \in X \).
- \( h(0) = 0 \).
- Controllability function \( L_c \) and observability function \( L_o \) smooth and exist.
Energy functions: Gramian extensions

\[ L_c(x_0) = \min_{u \in L_2(-\infty, 0)} \frac{1}{2} \int_{-\infty}^{0} \| u(t) \|^2 \, dt \]

\[ x(-\infty) = 0, \, x(0) = x_0 \]

Minimum amount of control energy necessary to reach state \( x_0 \). \( L_c \) is the so-called **controllability function**.
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\[ L_o(x_0) = \frac{1}{2} \int_{0}^{\infty} \| y(t) \|^2 \, dt, \quad x(0) = x_0, \quad u(\tau) = 0, \quad 0 \leq \tau < \infty \]

Output energy generated by state \( x_0 \). \( L_o \) is the so-called **observability function**
Nonlinear systems: balancing

- In linear case: $L_o(x) = \frac{1}{2} x^T M x$ and $L_c(x) = \frac{1}{2} x^T W^{-1} x$.

- Lyapunov and Hamilton-Jacobi-Bellmann equations characterize $L_o$ and $L_c$.

- Role of observability and controllability for linear systems is replaced by zero-state observability and asymptotic reachability (or anti-stabilizability).
Nonlinear systems: balancing

Lots of research efforts later (Fujimoto, Gray, Scherpen):

• Under appropriate conditions, there exists neighborhood $X$ of 0 and $x = \Phi(z)$ s.t.

$$L_c(\Phi(z)) = \frac{1}{2} \sum_{i=1}^{n} \frac{z_i^2}{\bar{\sigma}_i(z_i)} \quad L_o(\Phi(z)) = \frac{1}{2} \sum_{i=1}^{n} z_i^2 \bar{\sigma}_i(z_i).$$

In particular, on $X$, $\|\Sigma\|_H = \sup_{\Phi(z_1,0,\ldots,0) \in X} \bar{\sigma}_1(z_1)$.

• Singular value functions unique at coordinate axes.

• Tool for balanced structure preserving model reduction.
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- Singular value functions unique at coordinate axes.
- Tool for balanced structure preserving model reduction.
- Discrete time version similar! Fujimoto, Scherpen 2010.
Nonlinear gradient systems: balanced trunc.

Take $T(x) = G(x)^{-1}$, then balancing can be applied. Assume that $\sigma_k(x_k) \gg \sigma_{k+1}(x_{k+1})$, and split state (and matrices and functions) accordingly, i.e., $x^a = (x_1, \ldots, x_k)^T$, $x^b = (x_{k+1}, \ldots, x_n)^T$. Balanced truncation, $x^b = 0$, then
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**Proposition:** If

$$T^{ab}(x^a, 0) \frac{\partial P}{\partial x^b}(x^a, 0) = 0, \quad \text{and} \quad T^{ab}(x^a, 0) \frac{\partial h}{\partial x^b}(\bar{x}^a, 0) = 0,$$

then reduced order system is gradient system with pseudo-Riemannian metric $G^a(x^a) = T^{aa}(x^a, 0)^{-1}$. 
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**Main problem compared with linear case:** no specific structure for $G(x)$ is obtained!
Nonlinear gradient systems: sing. pert.

Also singular perturbation reduction different in nonlinear case. Restrictive assumption: in bal. form $x^b$ part of system linear.

**Proposition:** Under appropriate assumptions, reduced order system via singular perturbations is gradient again, i.e.,

$$\dot{x}^a = \hat{T}(x^a) \frac{\partial \hat{P}}{\partial x^a}(x^a) + \hat{T}(x^a) \frac{\partial \hat{h}}{\partial x^a}(x^a)u, \quad y = \hat{h}(x^a, u)$$

with $\hat{P}(x^a), \hat{h}(x^a, u)$ follow from solving $x^b$ from

$$\frac{\partial P}{\partial x^b}(x) + \frac{\partial h}{\partial x^b}(x)u$$
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\[
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\]

**Claim:** Via nonlinear Schur complement linearity assumption can be removed.
Nonlinear gradient systems: cross Gramian

Cross-Gramian for linear gradient systems uses symmetry property. How about nonlinear case?

- In Ionescu, Scherpen 2009 extension is given for prolongation and gradient extension → a **Sylvester** like equation is difficult to obtain.
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- In Ionescu, Scherpen 2009 extension is given for prolongation and gradient extension $\rightarrow$ a *Sylvester* like equation is difficult to obtain.

- In Ionescu, Fujimoto, Scherpen 2010/2011 other definition is given, i.e., observability and controllability for nonlinear system is "symmetrized", resulting in different cross-Gramian definition.
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- In Ionescu, Fujimoto, Scherpen 2010/2011 other definition is given, i.e., observability and controllability for nonlinear system is "symmetrized", resulting in different cross-Gramian definition.

- Again due to lack of structure in $G(x)$, no direct link with balancing is yet obtained.
Nonlinear gradient systems: cross Gramian

Result based on cross-Gramian definition for prolongations of gradient system:

**Corollary:** Take $f(x) = G(x)^{-1} \frac{\partial P}{\partial x}(x)$. Nonlinear cross Gramian $L(x)$ fulfills the following Sylvester like equation

$$p^T T(x) L(x) \frac{\partial f}{\partial x}(x) v + \frac{1}{2} p^T \frac{\partial h}{\partial x}(x) \frac{\partial^T h}{\partial x}(x) v =$$

$$-v^T \frac{\partial^2 L_p}{\partial v \partial x}(x, v) f(x) + \frac{\partial^T L_p}{\partial x}(x, v) f(x) - \bar{F}^T L(x) v,$$

with $\bar{F} = F - T(x) \dot{G}(x) v$, and $p$ and $v$ states of prolongations.
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Result based on cross-Gramian definition for prolongations of gradient system:

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p^T T(x) L(x) \frac{\partial f}{\partial x}(x) v + \frac{1}{2} p^T \frac{\partial h}{\partial x}(x) \frac{\partial^T h}{\partial x}(x) v = \\
- v^T \frac{\partial^2 L_o^p}{\partial v \partial x}(x, v) f(x) + \frac{\partial^T L_o^p}{\partial x}(x, v) f(x) - \bar{F}^T L(x) v,
\]

with \( \bar{F} = F - T(x) \dot{G}(x) v \), and \( p \) and \( v \) states of prolongations.

**Remark:** In linear case Sylvester equation obtained!
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- Nonlinear gradient systems: structure is less clear, more research is necessary to see if structure can be of some help for model order reduction. Nonlinear RLC circuits in Brayton-Moser form are gradient systems.
Concluding remarks

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- How to deal with DAE systems?