Coupled symbolic-numerical model reduction using the hierarchical structure of nonlinear electrical circuits

Model Reduction for Complex Dynamical Systems (ModRed 2010)
TU Berlin, Berlin, Germany, December 2-4, 2010

Oliver Schmidt
Patrick Lang
Outline

- Introduction
- Hierarchical Modelling and Model Reduction
- Implementations and Applications
Outline

- Introduction and Foundations
  - Analysis and Reduction Methods
  - Symbolic Techniques
- Hierarchical Modelling and Model Reduction
- Implementations and Applications
Analysis and Reduction Methods

- numerical techniques
  - parameters given as numerical values
  - applicable to very large systems
  - no qualitative insights
- symbolic techniques
  - symbolic parameters
  - not for too large circuits (complexity)
  - analytical description of functional relations and dependences (qualitative insights)
  - helpful in early stages of design process
Symbolic Techniques

original DAE

\[ \text{error function } E \]
\[ \text{error bound } \epsilon \]
\[ \text{numerical analysis } A \]

\[ y_F = A(F,u) \]
\[ E(y_F, y_G) < \epsilon \]
\[ y_G = A(G,u) \]
Symbolic Techniques

- algebraic manipulations
  \[ x = f(y) \]
  \[ 0 = g(x, y) \]
  \[ 0 = g(f(y), y) \]

- term cancellations
  \[ F_j : \sum_{i=1}^{N} t_i(x) = 0 \]
  \[ G_j : \sum_{k \neq i}^{N} t_i(\bar{x}) = 0 \]
Outline

- Introduction and Foundations
- Hierarchical Modelling and Model Reduction
  - Hierarchical Modelling
  - Hierarchical Model Reduction Algorithm
  - Subsystem Reduction
  - Subsystem Sensitivities
  - Subsystem Ranking
- Implementations and Applications
SyreNe – Subproject 5

- numerical efficiency
- applicability to large systems
- ...

- analytical insights
- parameterized behavioral models

Coupling

original DAE

reduced DAE
Hierarchical Structure

- hierarchical layout
- different subsystems, coupled by an interconnecting structure
Hierarchical Reduction

**idea:** exploitation of hierarchy
- reduce subsystems separately
- replace subsystems by reduced models

**advantages**
- faster processing of smaller problems
- coupling of different techniques
- recursive approach possible
  → level concept
- larger nonlinear circuits processable

\[
\begin{align*}
  f^0(x^1, y^1, x^2, y^2, x^3, y^3, x^4, y^4) &= 0 \\
  f^1(x^1, y^1, z^1) &= 0 \\
  f^2(x^2, y^2, z^2) &= 0 \\
  f^3(x^3, y^3, z^3) &= 0 \\
  f^4(x^4, y^4, z^4) &= 0
\end{align*}
\]
Hierarchical Reduction – Algorithm

**summary**

- choose reduction methods for separated subsystems
- compute several reduced models for each subsystem
- compute subsystem sensitivities
- hierarchical reduction by means of subsystem ranking and suitable replacements
- guaranteed accuracy by checking the accumulated error after each replacement
Subsystem Reduction – Workflow

- simulate subsystem in test bench (a), record voltage potentials at subsystem terminals
- connect subsystem terminals to voltage sources (b)
- setup of describing system of equations and reduction (c)
- removal of sources yields reduced subsystem (d)
Subsystem Sensitivities

- relation between errors of subsystem and entire system not available
- determine degree of reduction of subsystems by influence on entire system
  - simulate original system
  - replace $T_i$ by reduced system $T_{i,k}$
  - simulate “perturbed” entire system
- compute error on output of entire system
**Definition Subsystem Sensitivity**

- electrical circuit
  \[ \Sigma = ( \{ T_i | i = 1, \ldots, m \}, S ) \]
- reduction information \( r_{ij} \), e.g.
  \[ r_{ij} = "symbolic reduction, 10%" \] or
  \[ r_{ij} = "Arnoldi, 5 steps" \]
- error function \( E \)
- sensitivity of \( T_i \):

\[
\mathbf{s}_{T_i} = \left( (r_{i1}, E(y, y_{T_i}, r_{i1})), \ldots, (r_{im_T}, E(y, y_{T_i}, r_{im_T})) \right)
\]
Subsystem Ranking

- for each subsystem
  - order the entries of the sensitivity vector increasingly w.r.t. the error
- in each step of the hierarchical reduction
  - take the minimum of all first entries in the ordered sensitivity vectors
  - replace the respective subsystem by the corresponding model
  - check the accumulated error

```plaintext
forall subsystems $T_i$ do 
\[ L_i := \text{order}(s_{T_i}) \text{ w.r.t. } E(y, y_{T_i}; x_j) \]
\[ T_{i_0} := T_i \]
end 
\[ L := \{L_1, \ldots, L_n\} \]
\[ y := \mathcal{A}(\Sigma, u) \]
\[ \tilde{\Sigma} := \Sigma \]
repeat 
  compute $\left( r_k, E(y, y_{T_i}; x_j) \right) := \min_{i, L_i \in L} (\min(L_i))$ w.r.t. $E(y, y_{T_i}; x_j)$
  replace $T_{i_0}^{r_k}$ by $T_{i_0}^{r_k}$
  update($\tilde{\Sigma}$)
  $y_{\tilde{\Sigma}} := \mathcal{A}(\tilde{\Sigma}, u)$
  $\varepsilon_{\text{out}} := E(y, y_{\tilde{\Sigma}})$
  if $\varepsilon_{\text{out}} \leq \varepsilon$ then 
    $L_{i_0} := L_{i_0} \setminus \{\min(L_{i_0})\}$
    if $L_{i_0} = \emptyset$ then $L := L \setminus \{L_{i_0}\}$
  else 
    reset $T_{i_0}^{r_k}$ to $T_{i_0}^{r_k}$
    update($\tilde{\Sigma}$)
    $L := L \setminus \{L_{i_0}\}$
  end 
until $L = \emptyset$
```
Subsystem Ranking
Subsystem Ranking

The slide illustrates a subsystem ranking diagram with four subsystems labeled $T_1$, $T_2$, $T_3$, and $T_4$. The diagram shows the connections and dependencies between these subsystems, with $T_1$ and $T_2$ connected, $T_3$ and $T_4$ connected, and $T_1$ and $T_3$ also connected. The entire system is indicated at the top of the diagram.
Subsystem Ranking
Subsystem Ranking

The diagram shows a hierarchy of subsystems within an entire system. The subsystems are labeled $T_1$, $T_2$, $T_3$, and $T_4$. The connections and dependencies between these subsystems are indicated by the lines in the diagram.
Subsystem Ranking
Subsystem Ranking

entire system

$T_1$ $T_2$

$T_3$ $T_4$
Subsystem Ranking

The diagram illustrates the ranking of subsystems within an entire system. The subsystems are labeled as $T_1$, $T_2$, $T_3$, and $T_4$. The visual representation shows the interconnections and ranking within the system.
Subsystem Ranking

entire system

$T_1$  $T_2$

$T_3$  $T_4$
Subsytem Ranking

The diagram illustrates the subsystem ranking within the entire system. Each subsystem (T1, T2, T3, T4) is connected, indicating dependencies or interactions between them. The ranking is visually represented by the placement and connections of the subsystems within the entire system.
Subsystem Ranking

The diagram illustrates the ranking of subsystems within a system. The subsystems are ranked from the entire system down to individual components. Each subsystem is connected to its corresponding rank, indicating its relative importance or priority.
Subsystem Ranking

entire system

$T_1$  

$T_2$  

$T_3$  

$T_4$
Subsystem Ranking

entire system

$T_1$ $T_2$

$T_3$ $T_4$
Subsystem Ranking

The diagram illustrates the ranking of subsystems within an entire system. The subsystems are labeled as $T_1$, $T_2$, $T_3$, and $T_4$. Each subsystem is connected to the others, indicating their interdependencies within the system.
Subsystem Ranking

The diagram illustrates the ranking of subsystems within the entire system. Each subsystem is represented by a box labeled with a letter: $T_1$, $T_2$, $T_3$, and $T_4$. The subsystems are connected in a way that indicates their dependencies and interactions. The colors (green, red, and yellow) may represent different characteristics or categories of the subsystems. The diagram likely corresponds to a specific methodology or algorithm used in the analysis of the system's components.
Subsystem Ranking

entire system
Subsystem Ranking

The diagram illustrates the ranking of subsystems within an entire system. Subsystems $T_1$, $T_2$, $T_3$, and $T_4$ are shown interconnected, indicating their relationship within the system. The diagram emphasizes the hierarchical and interdependent nature of these subsystems.
Subsystem Ranking

etc. …
Outline

- Introduction and Foundations
- Hierarchical Modelling and Model Reduction
- Implementations and Applications
  - Implementations
  - Hierarchical Reduction of a Differential Amplifier
  - Hierarchical Reduction of an Operational Amplifier
Implementations

Hierarchical reduction algorithm has been implemented in ANALOGINSYDES

- **ReduceSubcircuits**
  - computation of reduced subsystem models
  - yields entire system with all reduced subsystem models appended
    (advantage: possibility for easy switching among different reduced models)

- **SensitivityAnalysis**
  - computes sensitivities of each subsystem
  - returns sensitivity vectors with entries ordered increasingly w.r.t. the error

- **HierarchicalReduction**
  - computes ranking in accordance with the subsystem sensitivities and performs subsystem replacements in the corresponding order
  - yields hierarchically reduced entire system with all reduced subsystem models appended
Example – Differential Amplifier

differential amplifier

- specification
  - discretized PDE transmission line models (20 line segments each)
  - sine wave excitation: 2 V, 100 kHz

- full system: 167 eq., 645 terms

- **non-hierarchical** symbolic reduction (*2h 11min*)
  - 3% error: 124 eq., 416 terms
Example – Differential Amplifier

differential amplifier

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  - discretized PDE transmission line models (20 line segments each)
  - sine wave excitation: 2 V, 100 kHz
  - full system: 167 eq., 645 terms
- **non-hierarchical** symbolic reduction (**2h 11min**)
  - 3% error: 124 eq., 416 terms
  - 10% error: 44 eq., 284 terms
Example – Differential Amplifier

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  - 10% error: 44 eq., 284 terms
- hierarchical coupled symbolic-numerical reduction (4min 50sec)
  - 3% error: 62 eq., 315 terms
Example – Differential Amplifier

differential amplifier

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  - 10% error: 44 eq., 284 terms
- hierarchical coupled symbolic-numerical reduction (4min 50sec)
  - 3% error: 62 eq., 315 terms
  - 10% error: 60 eq., 249 terms
Example – Differential Amplifier

differential amplifier

specification

- discretized PDE transmission line models (20 line segments each)
- sine wave excitation: 2 V, 100 kHz

cooperation with SyreNe-SP3, TU Berlin (T. Stykel, A. Steinbrecher)

- reduction of \textbf{L1} using \textbf{PABTEC} (BT), reduction of \textbf{L8}, \textbf{L9} using \textbf{Arnoldi}, \textbf{symbolic} reduction of \textbf{DUT}, \textbf{DUT2}

- full system: 191 eq., 695 terms
- hierarchically reduced system: time cost: \textbf{8min 20sec}

- 3\% error: 96 eq., 2114 terms
Example – Differential Amplifier

differential amplifier

- specification
  - discretized PDE transmission line models (20 line segments each)
  - sine wave excitation: 2 V, 100 kHz
- cooperation with SyreNe-SP3, TU Berlin (T. Stykel, A. Steinbrecher)
  - reduction of L1 using PABTEC (BT), reduction of L8, L9 using Arnoldi,
    symbolic reduction of DUT, DUT2
- full system: 191 eq., 695 terms
- hierarchically reduced system: time cost: **8 min 20 sec**
  - 3% error: 96 eq., 2114 terms
  - 10% error: 84 eq., 1190 terms
Example – Operational Amplifier

operational amplifier op741

- specification
- 7 subsystems
- symbolic reductions
- error bounds [%]
  \{2, 10, 20, 30, 50, 70, 90, 100\}
- 10% error (entire system)
- transient analysis
- \(L^2\) error function
- input:
  - sine wave excitation,
  - 0.8 V amplitude,
  - 1 kHz frequency,
  - \(T=\[0 \text{ s}, 0.002 \text{ s}\]
Example – Operational Amplifier

operational amplifier op741
- full system: 215 eq., 1050 terms
- non-hierarchical symbolic reduction
  - 10% permitted error
  - 97 eq., 593 terms
  - time cost: \( \sim 10.5 \text{h} \)
Example – Operational Amplifier

- **Operational amplifier op741**
  - full system:
    - 215 eq., 1050 terms
  - **non-hierarchical** symbolic reduction
    - 10% permitted error
    - 97 eq., 593 terms
    - time cost: ~10.5h
  - **hierarchical** reduction
    - 10% permitted error
    - 153 eq., 464 terms
    - time cost: 2h 22min
Earlier Results

compared to non-hierarchical approach

- significant savings in time for both
  - system reduction and
  - system simulation
- models with similar or better quality w.r.t.
  - number of equations and terms
  - error
Earlier Results

- further excitations (operational amplifier)
  - pulse
  - sine wave

- sum of three sine waves

<table>
<thead>
<tr>
<th>system</th>
<th>voltage pulse</th>
<th>3 kHz sine wave</th>
<th>sum of sine waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>215 / 1050</td>
<td>106 s</td>
<td>273 s</td>
<td>104 s</td>
</tr>
<tr>
<td>159 / 560</td>
<td>55 s</td>
<td>250 s</td>
<td>114 s</td>
</tr>
<tr>
<td>68 / 317</td>
<td>36.5 s</td>
<td>65 s</td>
<td>35.9 s</td>
</tr>
<tr>
<td>34 / 92</td>
<td>6.6 s</td>
<td>14.1 s</td>
<td>10.5 s</td>
</tr>
</tbody>
</table>
Earlier Results

- further excitations (differential amplifier)
  - pulse
  - sum of sine waves
  - sum of pulses
Thank you for your attention.