PABTEC: A software package for model reduction of nonlinear circuit equations

Tatjana Stykel and Andreas Steinbrecher

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SyreNe-Subproject 3:
Element-based model reduction in circuit simulation
PABTEC: A software package for model reduction of nonlinear circuit equations

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- Introduction, Motivation
- Model equations for electrical circuits with nonlinear elements
- Model order reduction
- MATLAB-Toolbox PABTEC
- Numerical tests
- Summary
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While the structural size of the electrical devices is decreasing, the complexity of the electrical circuits is increasing.
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Introduction

- While the structural size of the electrical devices is decreasing, the complexity of the electrical circuits is increasing.
- This leads to a system of model equations consisting up to millions or even more unknowns.
- Simulation of such large models is mostly impossible or, at least, unacceptably time and storage consuming.
- Model order reduction presents a way out of this dilemma.
- A general idea of model order reduction is to replace a large-scale system by a much smaller model which approximates the input-output relation of the large-scale system within a required accuracy.
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Model equations

\[ \mathcal{E}(x) \dot{x} = Ax + f(x) + Bu, \]
\[ y = B^T x, \]
Model equations

\[
\begin{align*}
\mathcal{E}(x) \dot{x} &= Ax + f(x) + Bu, \\
y &= B^T x,
\end{align*}
\]

**states** \( x = \begin{bmatrix} \eta^T & \nu^T_C & \nu^T_V \end{bmatrix}^T \)

**inputs** \( u = \begin{bmatrix} \nu^T_I & u^T_V \end{bmatrix}^T \)

**outputs** \( y = -\begin{bmatrix} u^T_I & \nu^T_V \end{bmatrix}^T \)

\( C, R, L, V, I \) as index denote conductors, resistors, inductors, voltage sources, current sources

\( \eta \) vector of node potentials

\( \nu_* \) vector of currents

\( u_* \) vector of voltages
Model equations

\[ \mathcal{E}(x) \dot{x} = Ax + f(x) + Bu, \]
\[ y = B^T x, \]

states \[ x = \begin{bmatrix} \eta^T & \xi_L^T & \nu_V^T \end{bmatrix}^T \]
inputs \[ u = \begin{bmatrix} \nu_I^T & u_Y^T \end{bmatrix}^T \]
outputs \[ y = - \begin{bmatrix} u_I^T & \nu_Y^T \end{bmatrix}^T \]

\[ \mathcal{E}(x) = \begin{bmatrix} A_C C(A_C \eta) A_C^T & 0 & 0 \\ 0 & \mathcal{L}(\nu_L) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]
\[ A = \begin{bmatrix} 0 & -A_L & -A_V \\ A_L^T & 0 & 0 \\ A_V^T & 0 & 0 \end{bmatrix}, \]
\[ f(x) = \begin{bmatrix} -A_R g(A_R \eta) \\ 0 \\ 0 \end{bmatrix}, \]
\[ B = \begin{bmatrix} -A_I & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix}. \]

\( C, R, L, V, I \) as index denote conductors, resistors, inductors, voltage sources, current sources
\( \eta \) vector of node potentials
\( \xi \) vector of currents
\( u \) vector of volatages
\( A \) incidence matrices
\( C \) conductance matrix-valued function
\( L \) inductance matrix-valued function
\( g \) resistor relation
We will assume that

(A1) the matrix $A_V$ has full column rank,
(A2) the matrix $[A_C, A_L, A_R, A_V]$ has full row rank,

The circuit does not contain loops of voltage sources and cutsets of current sources.
We will assume that

1. The matrix $A_V$ has full column rank,
2. The matrix $[A_C, A_L, A_R, A_V]$ has full row rank,
3. The circuit does not contain loops of voltage sources and cutsets of current sources.
4. The matrices $C(A_C^T \eta)$ and $L(\nu_L)$ are positive definite for all admissible $\eta$ and $\nu_L$, and
5. The function $g(A_R^T \eta)$ is monotonically increasing for all admissible $\eta$.

All circuit elements do not generate energy.
Furthermore, we assume without loss of generality that the circuit elements are ordered such that

\[ A_C = \begin{bmatrix} A_{\tilde{C}} & A_{\tilde{C}} \end{bmatrix}, \quad A_L = \begin{bmatrix} A_{\tilde{L}} & A_{\tilde{L}} \end{bmatrix}, \]
\[ A_R = \begin{bmatrix} A_{\tilde{R}} & A_{\tilde{R}} \end{bmatrix}, \]

We also assume that the linear and nonlinear elements are not mutually connected, i.e.,

\[ C(A_C^T \eta) = \begin{bmatrix} \tilde{C} & 0 \\ 0 & \tilde{C}(A_C^T \eta) \end{bmatrix}, \quad L(\tau_L) = \begin{bmatrix} \tilde{L} & 0 \\ 0 & \tilde{L}(\tau_{\tilde{L}}) \end{bmatrix}, \]
\[ g(A_R^T \eta) = \begin{bmatrix} \tilde{g} A_{\tilde{R}}^T \eta \\ \tilde{g}(A_{\tilde{R}}^T \eta) \end{bmatrix}, \]
Consequently, we have the model equations in the form

\[ \mathcal{E}(x) \dot{x} = Ax + f(x) + Bu, \]
\[ y = \mathcal{B}^T x, \]

with

\[ \mathcal{E}(x) = \begin{bmatrix} A \bar{\mathcal{C}} \bar{\mathcal{A}}^T \bar{\mathcal{C}} + A \bar{\mathcal{C}} \bar{\mathcal{C}}(A \bar{\mathcal{C}}^T \eta)A \bar{\mathcal{C}}^T \bar{\mathcal{C}} & 0 & 0 & 0 \\ 0 & \bar{\mathcal{L}} & 0 & 0 \\ 0 & 0 & \bar{\mathcal{L}}(\eta \bar{\mathcal{L}}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} -A \tilde{\mathcal{R}} \tilde{\mathcal{G}}(A \tilde{\mathcal{R}}^T \eta) \\ 0 \\ 0 \end{bmatrix}, \]

\[ \mathcal{A} = \begin{bmatrix} A \tilde{\mathcal{R}} \tilde{\mathcal{G}} \mathcal{A}^T \tilde{\mathcal{R}} & -A \bar{\mathcal{L}} & -A \tilde{\mathcal{L}} & -A \mathcal{V} \\ \mathcal{A}^T \bar{\mathcal{L}} & 0 & 0 & 0 \\ \mathcal{A}^T \tilde{\mathcal{L}} & 0 & 0 & 0 \\ \mathcal{A}^T \mathcal{V} & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} -A \mathcal{I} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \]
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nonlinear circuit equations
Model Order Reduction - Approach

Decoupling nonlinear circuit equations

linear subsystem

nonlinear subsystem

Decoupling nonlinear subsystem
Model Order Reduction - Approach

nonlinear circuit equations

Decoupling

linear subsystem

Model order reduction (e.g. Balanced Truncation)

nonlinear subsystem

reduced linear subsystem
Model Order Reduction - Approach

nonlinear circuit equations

linear subsystem

Decoupling

Model order reduction (e.g. Balanced Truncation)

nonlinear subsystem

reduced linear subsystem

Recoupling

reduced nonlinear system
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MATLAB-Toolbox: PABTEC

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MATLAB-Toolbox: PABTEC

\[
[Er,Ar,Br,Cr, \ldots ] = \text{PABTEC}(\text{inzidence matrices, parameter, } \ldots )
\]

Decoupling of linear subcircuits

\[
[ErI,ArI,BrI,CrI, \ldots ] = \text{PABTECL}(E,A,B,C, \ldots )
\]

Topology

Lyapunov
Preprocessing (Projectors)
Solving the Lyapunov equ. (ADI, Krylov methods)
Model reduction
Postprocessing

Riccati
Preprocessing (Projectors)
Solving the Riccati equ. (Newton method)
Model reduction
Postprocessing

Lure
Preprocessing (Projectors)
Solving the Lure equ. (Newton method)
Model reduction
Postprocessing

Linear

Nonlinear

Recoupling of the subcircuits

No L and C

No L or C

No CVI−loops

No LVI−cutsets

No R

Else

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MATLAB desktop keyboard shortcuts, such as Ctrl+5, are now customizable. In addition, many keyboard shortcuts have changed for improved consistency across the desktop.

To customize keyboard shortcuts, use Preferences. From there, you can also restore previous default settings by selecting "R2009a UNIX Default Set" from the 'Active settings' drop-down list. For more information, see Help.

Click here if you do not want to see this message again.
MATLAB desktop keyboard shortcuts, such as Ctrl+G, are now customizable. In addition, many keyboard shortcuts have changed for improved consistency across the desktop.

To customize keyboard shortcuts, use Preferences. From there, you can also restore previous default settings by selecting "R2009a UNIX Default Set" from the 'Active settings' drop-down list. For more information, see Help.

Click here if you do not want to see this message again.

>> addpath '/net/circuit/NetRed/Software/Pabtec/
>> pabtecgui
MATLAB-Toolbox: PABTEC - Graphical User Interface
MATLAB-Toolbox: PABTEC - Graphical User Interface

Problem specification
- Current directory
- Load system matrices from sys.mat
- Generate system matrices from netlist.m
- Load file manually

Batch processing
- Enable
- Add
- Delete

Computation
- Simulate original system
- Load instead
- Model reduction
- Load instead
- Simulate reduced system
- Load instead

System information
- Nodes: 22
- Voltage sources: 4
- Current sources: 0
- Resistors: 20
- Inductors: 0
- Capacitors: 0

System check:
- Dimensions: OK
- Symmetry: YES
- Index: 1
- Well posedness: YES
- Topology: (original) RCV
- (linear) RCV

System is: reducible: YES

START

Figure 8: System graph
MATLAB-Toolbox: PABTEC - Graphical User Interface
MATLAB-Toolbox: PABTEC - Graphical User Interface
MATLAB-Toolbox: PABTEC - Graphical User Interface

Problem specification
- current directory /net/circuit/ModRed/Examples/Linear/UI
- Load system matrices from sysdir...
- Generate system matrices from net...
- Load file manually

Batch processing
- Enable

Computation
- Simulate original system
- Model reduction
- Simulate reduced system

System information
- Nodes: 22
- Voltage sources: 4
- Current sources: 0
- Resistors: 20
- Inductors: 0
- Capacitors: 21
- System check: OK
- Symmetry: YES
- Index: 1
- Well posedness: YES
- Topology: RCV (original)
- System is: reducible

Parameters for the reduction
- Tolerance: 1.0e-04
- Order: 20
- Method: Lyap

Parameters for the (inner) LR-ADI iteration
- Max. iterations: 150
- Min. iterations: 1e-06
- Min. up: Off
- Min. values: 5
- Projector: 4
- Total: 3

ADI parameters
- # shifts: 20
- Largest FEV: 20
- Smallest FEV: 22
- Method: Heuristic...
- LU: Save
- Residual: Frobenius...
- Total: Iout 1

Parameters for the (outer) Newton iteration
- Max. iterations: 15
- Min. iterations: 1e-05
- Min. up: Off
- Min. values: 5

Time interval
- Initial time: 0 sec
- Final time: 0.01 sec

Original system
- Order of BDF-method: 2
- Max. Newton: 20

Reduced system
- Order of BDF-method: 2
- Max. Newton: 20

Export original system matrices
- Save in.mat
- Save in text

Export reduced system matrices
- Save matrices in .mat
- Save matrices in .txt

Export simulation data
- Save simulation data of original system...
- Save simulation data of reduced system...

Plots
- Show:
  - All
  - ADI
  - Linear
  - Hankel values
  - Frequency response
  - Absolute error
  - Relative error
- Save as:
  - *.eps
  - *.fig
  - *.png

START
MATLAB-Toolbox: PABTEC - Graphical User Interface

Problem specification:
- Current directory: [net/circuit/ModRed/Examples/Linear/LinCircl]
- Load system matrices from sysadm.mat
- Generate system matrices from netlist.m
- Load file manually

Batch processing:
- Enable
- Add
- Delete

Computation:
- Simulate original system
- Load instead
- Model reduction
- Load instead
- Simulate reduced system
- Load instead

System information:
- Nodes: 1007
- Voltage sources: 2
- Current sources: 0
- Resistors: 1006
- Inductors: 0
- Capacitors: 1005

System check:
- Dimensions: OK
- Symmetry: YES
- Index: 1
- Well posedness: YES
- Topology: RCV (original)
- RCV (linear)
- System is: reducible: YES

START

Figure 1: frequency_response
Frequency responses
- Full order
- PABTEC

Figure 2: ADI
Generalized LR-ADI iteration

Figure 3: characteristic_values
Hankel singular values

Figure 4: error
Absolute error and error bound

Figure 5: absolute_error
Absolute error and error bound

Figure 6: relative_error
Relative error

Simulation
- Input of the original System:
- Output of the original System:
- Original System:
r = 1005
r = 0.0001
BDF Order = 2
Sim Time = 0.0001

Statistics:
- [Values]
- [Values]
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Example 01 - Problem

State dimension of the model equations $n = 1503$

Simulation is done for $t \in [0s, 0.07s]$ using BDF method of order 2 with fixed stepsize of length $1 \cdot 10^{-5}$. The computations are done with MATLAB.
Example 01 - Simulation results

\textit{Input: voltage source}

\begin{align*}
\Delta V & \leq 10^{-2} \\
\Delta V & \leq 10^{-5}
\end{align*}
Example 01 - Efficiency

<table>
<thead>
<tr>
<th>Dimension of the original system</th>
<th>1503</th>
<th>1503</th>
<th>1503</th>
<th>1503</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time for the original system</td>
<td>24012s</td>
<td>24012s</td>
<td>24012s</td>
<td>24012s</td>
</tr>
<tr>
<td>Prescribed tolerance for the model reduction</td>
<td>1e-02</td>
<td>1e-03</td>
<td>1e-04</td>
<td>1e-05</td>
</tr>
<tr>
<td>Time for the model reduction</td>
<td>15s</td>
<td>24s</td>
<td>42s</td>
<td>61s</td>
</tr>
<tr>
<td>Dimension of the reduced system</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>Simulation time for the reduced system</td>
<td>82s</td>
<td>110s</td>
<td>122s</td>
<td>155s</td>
</tr>
<tr>
<td>Obtained error of the output of the reduced system</td>
<td>7.0e-06</td>
<td>6.2e-07</td>
<td>2.0e-07</td>
<td>4.2e-07</td>
</tr>
<tr>
<td>Speedup</td>
<td>294.0</td>
<td>219.0</td>
<td>197.4</td>
<td>155.0</td>
</tr>
</tbody>
</table>
Example 02 - Problem

3001 nodes 1 voltage source
2000 linear capacities 1 output
1990 linear resistors 10 diode
991 linear inductors 10 nonlinear inductors

State dimension of the model equations $n = 4003$

Simulation is done for $t \in [0s, 0.05s]$ using BDF method of order 2 with fixed stepsize of length $5 \cdot 10^{-5}$. The computations are done with MATLAB.
Example 02 - Simulation results

Input: voltage source

Output: negative current of the voltage source

Error of the output

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Example 02 - Efficiency

### Dimension of the red. system vs. prescribed tolerance

<table>
<thead>
<tr>
<th>prescribed tolerance</th>
<th>Dimension of the red. system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-8}$</td>
<td>340</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>280</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>160</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>20</td>
</tr>
</tbody>
</table>

### Error of the output vs. prescribed tolerance

<table>
<thead>
<tr>
<th>prescribed tolerance</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-8}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$10^{-0}$</td>
</tr>
</tbody>
</table>

### Speedup vs. prescribed tolerance

<table>
<thead>
<tr>
<th>prescribed tolerance</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-8}$</td>
<td>107.2</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>68.5</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>36.5</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>16.5</td>
</tr>
</tbody>
</table>

| dimension of the original system | 4003 | 4003 | 4003 | 4003 |
| simulation time for the original system | 4557s | 4557s | 4557s | 4557s |
| prescribed tolerance for the model reduction | 1e-03 | 1e-05 | 1e-07 | 1e-09 |
| time for the model reduction | 902s | 822s | 834s | 900s |
| dimension of the reduced system | 149 | 203 | 260 | 329 |
| simulation time for the reduced system | 43s | 67s | 125s | 277s |
| obtained error of the output of the red. system | 1.6e-04 | 4.4e-06 | 1.16e-06 | 1.9e-06 |
| speedup | 107.2 | 68.5 | 36.5 | 16.5 |
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We presented a model order reduction approach for the model equations of nonlinear circuits.
We presented a model order reduction approach for the model equations of nonlinear circuits.

The model reduction technique bases on ...

- decoupling of linear and nonlinear subcircuits
- model reduction of the remained linear part
- recoupling of the reduced linear subcircuit with the unchanged nonlinear subcircuit
We presented a model order reduction approach for the model equations of nonlinear circuits.

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We presented the MATLAB-Toolbox PABTEC for the model order reduction of model equations of nonlinear circuits based on the technique above.
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The model reduction technique bases on ...
- decoupling of linear and nonlinear subcircuits
- model reduction of the remained linear part
- recoupling of the reduced linear subcircuit with the unchanged nonlinear subcircuit

We presented the MATLAB-Toolbox PABTEC for the model order reduction of model equations of nonlinear circuits based on the technique above.

The efficacy and applicability of the proposed model reduction approach was demonstrated on several numerical examples.
Thank you for your attention.