Detecting extended strongly hyperbolic matrix polynomials

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A Hermitian matrix polynomial

$$P(\lambda) = \sum_{j=0}^{\ell} \lambda^j A_j$$

is extended strongly hyperbolic if there exists $\mu \in \mathbb{R}$ such that $P(\mu)$ is positive definite and for every $x \in \mathbb{C}^n$ the scalar equation $x^H P(\lambda)x = 0$ has ℓ distinct zeros in $\mathbb{R} \cup \{\infty\}$.

In [1] it was proved that $P(\lambda)$ is extended strongly hyperbolic if and only if there exist $\gamma_j \in \mathbb{R} \cup \{\infty\}$ with $\gamma_0 > \gamma_1 > \cdots > \gamma_{\ell-1}$ ($\gamma_0 = \infty$ being possible) such that $(-1)^j P(\gamma_j) > 0$ for $j = 0, \ldots, \ell - 1$, and each of the intervals (γ_j, γ_{j-1}), $j = \ell - 1, \ldots, 0$ and $(-\infty, \gamma_{\ell-1}) \cup (\gamma_0, \infty]$ contains exactly n eigenvalues.

Taking advantage of a generalization of the variational principle for nonlinear eigenvalue problems in [2] we characterize all eigenvalues of $P(\lambda)$ as minmax values of ℓ Rayleigh functionals, and we discuss a quadratically convergent method for computing all eigenvalues in each of the intervals (γ_j, γ_{j-1}) and $(-\infty, \gamma_{\ell-1}) \cup (\gamma_0, \infty]$ in a systematic way. In particular this method converges to the extreme eigenvalues in the intervals globally and monotonically. Conversely, if one of these iterations looses monotonicity then the corresponding matrix polynomial is not extended strongly hyperbolic.

Since extended strongly hyperbolic is a generalization of hyperbolic and overdamped the procedure applies also to these types of eigenpoblems.

References

[1] N.J. Higham, D.S. Mackey, and F. Tisseur. Notes on hyperbolic matrix polynomials and definite linearizations. Technical Report MIMS EPrint: 2007.97, School of Mathematics, The University of Manchester, 2007.

[2] H. Voss and B. Werner. A minimax principle for nonlinear eigenvalue problems with applications to nonoverdamped systems. *Math. Meth. Appl. Sci.*, 4:415–424, 1982.