Structured Pseudospectra and the condition of a nonderogatory eigenvalue

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Abstract. Let λ be a nonderogatory eigenvalue of $A \in \mathbb{C}^{n\times n}$ of algebraic multiplicity m. The sensitivity of λ with respect to matrix perturbations of the form $A \rightsquigarrow A + \Delta, \Delta \in \boldsymbol{\Delta}$, is measured by the structured condition number $\kappa_{\boldsymbol{\Delta}}(A, \lambda)$. Here $\boldsymbol{\Delta}$ denotes the set of admissible perturbations. However, if $\boldsymbol{\Delta}$ is not a vector space over $\mathbb{C}$ then $\kappa_{\boldsymbol{\Delta}}(A, \lambda)$ provides only incomplete information about the mobility of λ under small perturbations from $\boldsymbol{\Delta}$. The full information is then given by the set $K_{\boldsymbol{\Delta}}(x, y) = \{y^*\Delta x;\ \Delta \in \boldsymbol{\Delta}, \|\Delta\| \leq 1\} \subset \mathbb{C}$ which depends on $\boldsymbol{\Delta}$, a pair of normalized right and left eigenvectors x, y and the norm $\|\cdot\|$ which measures the size of the perturbations. We always have $\kappa_{\boldsymbol{\Delta}}(A, \lambda) = \max\{|z|^{1/m};\ z \in K_{\boldsymbol{\Delta}}(x, y)\}$. Furthermore, $K_{\boldsymbol{\Delta}}(x, y)$ determines the shape and growth of the $\boldsymbol{\Delta}$-structured pseudospectrum in a neighbourhood of λ. In this talk we study the sets $K_{\boldsymbol{\Delta}}(x, y)$ and obtain methods for computing them. In doing so we obtain explicit formulae for structured eigenvalue condition numbers with respect to many important perturbation classes.