Simulation and optimal control of the hydration of concrete for avoiding cracks

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Concrete

Construction material
Various, very different uses
But basic principles for all uses

Hydration

Cement + aggregate + admixtures + water/ice
Concrete (structure + strength) + heat
Plastic/viscous mass
Solid

Modelling of the hydration

Rate of the chemical reaction depends on the temperature $T$. Therefore replace time by the

Maturity

Common form

$$\tau_w(x,t) = \int g(y(x,\theta)) d\theta.$$  
Most popular choice of $g$ after Freihsleben
Hansen $g(y(x,t)) = e^{-(x - \text{maturity})}$, but others also possible.
Note $\tau_w = t$ if constant temperature $T = 20$ C.

Measure advance of hydration by

Degree of hydration

$$\alpha(x,\tau_w(t),t) = \frac{Q(\tau_w(t),t)}{Q_\infty} = \int_0^{\tau_w} h(\theta) d\theta.$$ 
Common shape of the development of $\alpha$ for all different concrete recipes.
Widely used: Model of Jonasson

$$\alpha = a_0 \log \left(1 - \frac{\alpha(t)}{\alpha_0}\right)^{\delta},$$ 
with $a_0, \alpha_0 < 0$, $t_k > 0$.
Often $a_0 = -1$.

Experimental data of Prof. M. Keuser and Dr.-Ing. E. Hailer, UnIW München

Heat equation

The development of mechanical properties in every point depends on the temperature:
Solve the heat equation with the currently produced heat as source term

$$c_p \frac{\partial \psi}{\partial t} - \lambda \frac{\partial^2 \psi}{\partial y^2} = Q_0 \frac{\partial \alpha}{\partial \tau_w} \frac{\partial \tau_w}{\partial t}$$

State equation

As $\tau_w$ can be computed additional and Robin boundary conditions are appropriate the system is

$$\begin{align*}
\tau_w &= g(y) \\
c_p \frac{\partial \psi}{\partial t} - \lambda \frac{\partial^2 \psi}{\partial y^2} &= Q_0 \frac{\partial \alpha}{\partial \tau_w} \frac{\partial \tau_w}{\partial t}
\end{align*}$$

Uniqueness of the solution: direct (all the parameter have the “right” sign).
Existence: Use Schauder’s fixed point theorem

Simulation

Two dimensional structure: New wall on old plate. FEM in space, ODE integration scheme in time

Modelling of mechanics

Young modulus $E = \frac{\alpha - \alpha_0}{1 - \alpha_0}$
tensile strength $f_{ct}(\alpha(\tau_w(t))) = f_{ct,\infty} \left(\frac{\alpha - \alpha_0}{1 - \alpha_0}\right)^{\gamma_3}$
thermal strain $\alpha_{\text{therm}}(x, t) = \alpha_{\text{therm}}(y(x), t_0)^{b_3}$
with $\gamma_3 = \frac{1}{2}$, $\gamma_3 = 1$ and $y(x, t_0) = y(x, t_1) - y(x, t_0)$.

Linear elastic material law

For one time step

$$\sigma(x, t_0) = \sigma(x, t_1) + A(E(t_0), t_1): \alpha_{\text{therm}}(y(x), t_0)^{b_3}$$

Underlying continuous equation

$$\dot{\alpha}(x, t) = A(E(t), t) : \alpha_{\text{therm}}(y(x), t).$$

The fourth order tensor $A$ depends in the usual way on Young modulus $E(t)$ and Poisson ratio $\nu$.
Even in linear model:
Time dependent Young modulus.

Viscoelastic material law

Stress in the $n$-th time step

$$\sigma(x, t_n) = \sum_{k=1}^n A(E(t_k), t_k) : \alpha_{\text{therm}}(y(x), t_{k-1}) \psi(t_{k-1}).$$

Underlying continuous equation

$$\dot{\alpha}(x, t) = A(E(t), t) : \alpha_{\text{therm}}(y(x), t) + \int_0^t A(E(t), t) : \alpha_{\text{therm}}(y(x), t) \psi(t - \theta) d\theta.$$

Mechanical Crack Criterion

No cracks, if for all $x$ and $t$

$$\min_{0 \leq t \leq T} |(0, \sigma(x, t))| < I_2(x, \sigma(\tau_w(t), t))$$

In engineering practice even

$$\min_{0 \leq t \leq T} |(0, \sigma(x, t))| < I_2(x, \sigma(\tau_w(t), t)) \leq k = 1.$$  

- Mechanical justification
- Additional computation of $\alpha_c$, $f_{ct}$, $E_{\text{therm}}$, $\sigma$.

Temperature Crack Criterion

New wall on top of old bottom plate:
Engineering experience:
No cracks if the temperature difference between the middle points of wall and basement is less than 15 K:

$$|y(x_{\text{mid}, t}) - y(x_{\text{mid}, t})| < 15K \forall t$$

- Easy to check
- Still in use by civil engineers
- No mechanical justification

Controls

The crack risk can be influenced by

Possible choices

- Influence on
- change concrete recipe $\lambda$, $c$, $\nu$, $Q$.
- fresh concrete temperature $a_2$, $b_2$, $r_2$, $f_{ct,\infty}$, $E_{\infty}$.
- cooling pipes $\sigma_{\text{bnd}}$, $\sigma_{\text{wind}}$.

Optimal Control Problems

Minimize $J(y, \tau, u)$ includes costs for the control $u$ (ODE + PDE).
S.t.: state equation
influenced by control $u$
and crack criterion
state constraint

Cooperation with civil engineers

- R. Nothnagel (iBMB, TU Braunschweig)
- L. Nieter (Bilfinger Berger)
- E. Hailer and M. Keuser (UnIW München)

References