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Infinite Dimensional Operator Matrices
Theory and Applications

TU WIEN

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References to solutions of the problems

See next page for bibliographical references.

1) [B74, Lemma I.2.1]

2) [B74, Lemma I.6.2]. Note that in the book by Bognár 'non-negative' subspaces are called 'positive' and 'positive' subspaces 'positive definite'. See [B74, Remark 6.5] for the fact that $\mathcal{M}^{[\perp]}$ need not be maximal non-positive.

3) [DL96, Lemma 1.2]

4) [L82, Proposition 1.2]

5) [B74, Theorem V.1.3.] or [AI89, §1.6.13]

6) [IKL82, Theorem 2.4]

7) [L82, Proposition 5.1]

8) [L82, Theorem 5.2] and its proof.

9) [L82, p. 9]

10) [L82, Proposition 3.1]

11) The proof is essentially the same as in the Hilbert space case.

12) [L82, Proposition 3.2]

13) [B74, VII.1.3]

14) The operator A is self-adjoint in the Pontryagin space $L^2_\sigma \oplus \mathbb{C}$ and hence has an eigenvalue in the closed upper half-plane. Simplifying the eigenvalue equation for A one gets relation (1). The condition $\int_{\mathbb{R}} \frac{d\sigma(t)}{|t-z_0|^2} \leq 1$ is equivalent to the fact that the eigenvector is non-positive.

15) One possibility is to transform the upper half-plane to the unit disc with a Möbius transform and apply results by Denjoy [D26] and Wolff [W26] (see also [B81]).

16) [A79, Theorem 5.1]

17) [A79, Theorem 5.2]

18) [L82, pp. 12,13]

References

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