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References to solutions of problems

1. This follows basically from the definition. For quadratic forms see, e.g. [Kato95].

2. $\sigma(T) = \left\{\frac{1}{n} \colon n \in \mathbb{N}\right\}$. The variational principle cannot be applied because T is not continuous in norm resolvent sense at 0. It can be applied to the function $S(\lambda) = -T(-\lambda)$ on the interval $[-3, 0)$.

3. A similar example was considered in [BEL00, Example 3.6].
   $\sigma(T) = \{0, 1\}$, $\kappa = 0$. The variation gives the eigenvalue 1 and not the point 0 in the essential spectrum.

4. See [EL04, Section 3.2].

5. See [EL04, pp. 296–297].

References

[BEL00] Binding, P., Eschwé, D., Langer, H., Variational principles for real eigenvalues of self-adjoint operator pencils. *Integral Equations Operator Theory* **38** (2000), 190–206.

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[Kato95] Kato, T., *Perturbation Theory for Linear Operators*. Reprint of the 1980 edition. Classics in Mathematics. Springer-Verlag, Berlin, 1995.