

References to solutions of the problems

See last page for bibliographical references.

1. Unbounded operators.

- i) simple, [Kat95], p.164, Problem III.5.6
- ii) simple, [Kat95], p.164, Problem III.5.7
- iii) [GGK90], Prop. XVII, p.374
- iv) simple, [MV92], Lemma 19.8
- v) simple, [MV92], Lemma 19.9

2. Relative boundedness.

- i) [Tre08], Remark 2.1.3 i), p.92; [Kat95], Remark IV.1.5, p.191
- ii) [Tre08], Lemma 2.1.6
- iii) [Tre08], Remark 2.1.3 ii), p.92; [Kat95], §V.4.1, p.287

3. Relative compactness and relative bound 0.

- i) [EE87], Corollary III.7.7, p.135
- ii) [Tre08], Proposition 2.1.19, p.97
- iii) [Tre08], Corollary 2.1.20, p.97

4. Classical perturbation theorems.

- i) [Kat95], Theorems IV.1.1 and IV.1.16, p. 190/196
- ii) [Kat95], Theorem V.4.11, p.291
- iii) [Tre08], Theorem 2.1.13, p. 95; [GGK90], Theorem XVII.4.3, Corollary XVII.4.4, p.377

5. Useful inequalities.

- i) [Wei03], Satz 17.1, p.168
- ii) [KZ03], Example 1
- iii) simple: partial integration

6. Schrödinger operators in \mathbb{R}^n ($n \geq 3$).

- i) use Hardy's inequality; [Wei03], Satz 17.2 a), p.169
- ii) approximate V by C_0^∞ functions and use the compactness of the embedding $W_2^2(\Omega) \hookrightarrow W_2^1(\Omega)$ for bounded Ω ; alternatively [Wei03], Satz 17.2 b), p.169
- iii) [Wei03], Satz 17.10 a), p.179

7. Dirac operators.

- i) [Tha92], Theorem 1.1
- ii) [Wei03], Satz 20.3 a), p.267
- iii) [Tha92], Section 4.3.4, p.114-117

8. Klein-Gordon equation.

[LNT06]

Quiz.

It is an ellipse ...

9. Simple properties of the quadratic numerical range (QNR).

- i) easy
- ii) [Tre08], Prop. 1.1.7 i), p.6, for unbounded \mathcal{A}
- iii) [Tre08], Prop. 1.1.7 ii), p.6, for unbounded \mathcal{A}
- iv) [Tre08], Prop. 1.1.12, p.7, for unbounded \mathcal{A}

10. Self-adjoint and \mathcal{J} -self-adjoint block operator matrices.

- i) [Tre08], Prop. 2.6.1, p.143
- ii) [Tre08], Thm. 2.6.5, p.145, there ass. missing which imply that S^* is T^* -compact
- iii) [Tre08], Thm. 2.6.6, p.145
- iv) [Tre08], Prop. 2.6.1 and Thm. 2.6.6

11. Block operator matrices with real QNR.

- i) easy
- ii) [Tre08], Lemma 1.1.16, p.9
- iii) [Tre08], Thm. 2.6.7, p.146

12. Spectral inclusion.

- i) [Tre08], Lemma 2.5.16, p.136
- ii) [Tre08], Thm. 2.5.12, p.134
- iii) [Tre08], Example 2.5.14, p.135

13. \mathcal{A} -invariant subspaces and Riccati equations.

- i) [AMS09], Lemma 2.6, Remark 2.7
- ii) easy

14. Schur complements and spectrum.

- i) [Tre08], Lemma 2.3.2, p.111
- ii) [Tre08], proof of Thm. 2.2.14, p. 105
- iii) [Tre08], Prop. 2.3.4, p.113

15. Essential spectrum.

- i) [GGK90], Thm. XVII.3.1, p.374
- ii) [Tre08], Thm. 2.1.15, p.95
- iii) [Tre08], Cor. 2.4.13, p.125

16. Schur complements and QNR.

- i) [Tre08], Lemma 2.5.7, p.131
- ii) [Tre08], Prop. 2.6.10, p.149
- iii) analogous to [Tre08], Lemma 2.6.9., p.148
- iv) [Tre08], Lemma 2.6.9.

References

- [AMS09] Albeverio, S., Motovilov, A. K., Shkalikov, A. A. Bounds on variation of spectral subspaces under J -self-adjoint perturbations. *Integral Equations Operator Theory*, 64(4):455–486, 2009.
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