Thursday, December 13th

9:00 – 9:20	*Opening*

Chair: Branko Curgus

9:20 – 9:45	**Aad Dijksma** The Schur transformation for Nevanlinna functions: Operator representations, resolvent matrices, and orthogonal polynomials
9:45 – 10:10	**Paul Binding** Two parameter eigencurves for non-definite eigenproblems
10:10 – 10:35	**Andrei Shkalikov** Dissipative operators in Krein space. Invariant subspaces and properties of restrictions
10:35 – 11:45	*Refund of travel expenses (MA 674)* & *Coffee break (DFG Lounge MA 315)*

Thursday, December 13th

Chair: Vadim Adamyan

11:45 – 12:10	**Tomas Azizov** On the Ando-Khatskevich-Shulman theorem
12:10 – 12:35	**Daniel Alpay** Rigidity, boundary interpolation and reproducing kernels
12:35 – 13:00	**Lev Sakhnovich** J-theory and random matrices
13:00 – 14:45	*Lunch break*

Thursday, December 13th

Chair: Andreas Fleige

14:45 – 15:10 **Hagen Neidhardt**
On trace formula and Birman-Krein formula for pairs of extensions

15:10 – 15:35 **Annemarie Luger**
More on the operator model for the hydrogen atom

15:35 – 16:00 **Ilia Karabash**
Forward-backward kinetic equations and the similarity problem for Sturm-Liouville operators

16:00 – 16:25 **Natalia Rozhenko**
Passive impedance bi-stable systems with losses of scattering channels

16:25 – 17:00 *Coffee break (DFG Lounge MA 315)*

Thursday, December 13th

Chair: Kresimir Veselic

17:00 – 17:25 **Ekaterina Lopushanskaya**
Moment problems for real measures on the unit circle

17:25 – 17:50 **Kerstin Günther**
Unbounded operators on interpolation spaces

17:50 – 18:15 **Aleksey Kostenko**
The similarity problem for J-nonnegative Sturm-Liouville operators

18:15 – 18:40 **Rudi Wietsma**
Products of Nevanlinna functions with certain rational functions

Friday, December 14th

Chair: Harm Bart

9:00 – 9:25	**Jean-Philippe Labrousse** Bisectors and isometries on Hilbert spaces
9:25 – 9:50	**Branko Curgus** Eigenvalue problems with boundary conditions depending polynomially on the eigenparameter
9:50 – 10:15	**Andre Ran** Inertia theorems based on operator Lyapunov equations
10:15 – 10:40	**Leiba Rodman** Canonical structures for palindromic matrix polynomials
10:40 – 11:30	*Conference photo* & *Coffee break (MA 366)*

Friday, December 14th

Chair: Vladimir Derkach

11:30 – 11:55	**Volker Mehrmann** Structured matrix polynomials: Linearization and condensed forms
11:55 – 12:20	**Birgit Jacob** Interpolation by vector-valued analytic functions with applications to controllability
12:20 – 12:45	**Olaf Post** First order operators and boundary triples
12:45 – 14:30	*Lunch break*

Friday, December 14th

Chair: Seppo Hassi

14:30 – 14:55 **Vyacheslav Pivovarchik**
Inverse spectral problems for Sturm-Liouville equation on trees

14:55 – 15:20 **Andreas Fleige**
The Riesz basis property of indefinite Sturm-Liouville problems with a non odd weight function

15:20 – 15:45 **Mikhail Denisov**
On numbers of negative eigenvalues of some products of selfadjoint operators

15:45 – 16:10 **Mark-Alexander Henn**
Hyponormal and strongly hyponormal matrices in inner product spaces

16:10 – 17:00 *Coffee break (DFG Lounge MA 315)*

Friday, December 14th

Chair: Franciszek H. Szafraniec

17:00 – 17:25 **Michael Dritschel**
Schwarz-Pick inequalities via transfer functions

17:25 – 17:50 **Adrian Sandovici**
Invariant nonnegative relations in Hilbert spaces

17:50 – 18:15 **Maxim Derevjagin**
A Jacobi matrices approach to Nevanlinna-Pick problems

18:15 – 18:40 **Anton Kutsenko**
Borg type uniqueness theorems for periodic Jacobi operators with matrix valued coefficients

18:40 – 19:00 **Karl-Heinz Förster**
GAMM activity group *"Applied Operator Theory"*

20:00 *Conference dinner*

Restaurant Cortez, Uhlandstr. 149, 10719 Berlin

Saturday, December 15th

Chair: Rostyslav Hryniv

9:00 – 9:25 **Harm Bart**
Vector-valued logarithmic residues and non-commutative Gelfand theory

9:25 – 9:50 **Seppo Hassi**
On passive discrete-time systems with a normal main operator

9:50 – 10:15 **Vladimir Derkach**
On the uniform convergence of Pade approximants for a class of definitizable functions

10:15 – 10:40 **Marina Chugunova**
Spectral properties of the J-self-adjoint operator associated with the periodic heat equation

10:40 – 11:30 *Coffee break (DFG Lounge MA 315)*

Saturday, December 15th

Chair: Aad Dijksma

11:30 – 11:40 **Aurelian Gheondea**
Peter Jonas - friend and collaborator

11:40 – 12:05 **Aurelian Gheondea**
When are the products of two normal operators normal ?

12:05 – 12:30 **Franciszek H. Szafraniec**
A look at Krein space:
New thoughts and old truths

12:30 – 12:55 **Andras Batkai**
Polynomial stability:
Some recent results and open problems

12:55 – 14:30 *Lunch break*

Saturday, December 15th

Chair: Andrei Shkalikov

14:30 – 14:55 **Kresimir Veselic**
Perturbation bounds for relativistic spectra

14:55 – 15:20 **Victor Khatskevich**
The KE-problem: Description of diagonal elements

15:20 – 15:45 **Matej Tusek**
On spectrum of quantum dot with impurity in Lobachevsky plane

15:45 – 16:10 **Uwe Günther**
Projective Hilbert space structures at exceptional points and Krein space related boost deformations of Bloch spheres

16:10 – 16:50 *Coffee Break (DFG Lounge MA 315)*

Saturday, December 15th

Chair: Paul Binding

16:50 – 17:15 **Igor Sheipak**
Asymptotics of eigenvalues of a Sturm-Liouville problem with discrete self-similar indefinite weight

17:15 – 17:40 **Michal Wojtylak**
Commuting domination in Pontryagin spaces

17:40 – 18:05 **Qutaibeh Katatbeh**
Complex eigenvalues of indefinite Sturm-Liouville operators

Sunday, December 16th

Chair: Leiba Rodman

9:00 – 9:25 **Vadim Adamyan**
Local perturbations on absolutely continuous spectrum

9:25 – 9:50 **Yury Arlinskii**
Iterates of the Schur class operator-valued function and their conservative realizations

9:50 – 10:15 **Sergei G. Pyatkov**
On the Riesz basis property in elliptic eigenvalue problems with an indefinite weight function

10:15 – 10:40 **Yuri Shondin**
On realizations of supersymmetric Dirac operator with Aharanov-Bohm magnetic field

10:40 – 11:20 *Coffee break (DFG Lounge MA 315)*

Sunday, December 16th

Chair: Henk de Snoo

11:20 – 11:45 **Rostyslav Hryniv**
Reconstruction of the Klein-Gordon equation

11:45 – 12:10 **Matthias Langer**
The Virozub-Matsaev condition and spectrum of definite type for self-adjoint operator functions

12:10 – 12:35 **Christian Mehl**
Singular-value-like decompositions in indefinite inner product spaces

12:35 – 13:00 **Vladimir Strauss**
On Spectralizable Operators

13:00 – 14:30 *Lunch break*

Sunday, December 16th

Chair: Andre Ran

14:30 – 14:55	**Lyudmila Sukhotcheva** On reducing of selfadjoint operators to diagonal form
14:55 – 15:20	**Konstantin Pankrashkin** Applications of Krein resolvent formula to localization on quantum graphs
15:20 – 15:45	**Jussi Behrndt** Compact and finite rank perturbations of linear relations
15:45	*Closing*

In Memory of Peter Jonas (1941 - 2007)

Peter Jonas was born on July 18th, 1941 in Memel, now Klaipeda, at that time the most eastern town of East Prussia. After the war he moved with his mother and grandmother to Blankenfelde - a small village near Berlin, where he lived until the end of his school education.

In 1959 he started to study mathematics at the Technische Universität Dresden. Here he met Heinz Langer, who was teaching exercise classes in analysis at that time, and Peter wrote his diploma thesis on stability problems of infinite dimensional Hamiltonian systems under the supervision of Heinz Langer.

After his diploma in 1964 Peter Jonas got a position at the Karl-Weierstrass Institute of the Academy of Sciences in East Berlin where he first worked with his PhD supervisor Josef Naas on problems in differential geometry, partial differential equations and conformal mappings. In this time he married his wife Erika and his children Simon and Judith were born. After his PhD in 1969 Peter joined the mathematical physics group around Hellmut Baumgärtel, and self-adjoint and unitary operators in Krein spaces became the main topic of his research. These activities culminated in the cooperation with Mark Krein and Heinz Langer; both had much influence on his Habilitation thesis *"Die Spurformel der Störungstheorie für einige Klassen unitärer und selbstadjungierter Operatoren im Kreinraum"*, (1987).

Peter Jonas established fruitful scientific contacts with many mathematicians in the Soviet Union and other Eastern European countries, many of these colleagues became close personal friends, among them Tomas Azizov, Branko Curgus, Aurelian Gheondea and Vladimir Strauss. At conferences in Eastern Europe he also met with West European colleagues, but at that time it was impossible for him to visit them in their home countries or West Berlin.

The political changes in 1989 had a tremendous influence on Peters life. The Karl-Weierstrass Institute was closed down in 1991, Peter lost his permanent position and became a member of the so-called Wissenschaftler-Integrations-Programm; a program that tried to incorporate employees of scientific institutions in East Germany into universities. However, it turned out that this program was rather inefficient and, as a result, Peters situation was vague. But it was not Peters habit to complain, rather he used this situation to obtain various positions at the Technische Universität Berlin, Freie Universität Berlin and at the Universität Potsdam. After a research stay in Bellingham (USA) he finally settled down at the Technische Universität where he worked until his retirement in 2006. In his last years Peter Jonas used the possibility to meet and discuss with his colleagues and friends in the USA, Israel, Austria, Venezuela, Turkey and the Netherlands. Beside his passion for mathematics, Peter was very interested in Asian culture, in particular, Buddhism.

The Functional Analysis group here at the Technische Universität Berlin has greatly benefited from Peter. With tremendous patience he instructed and supervised PhD and diploma students, he gave courses and special lectures in operator theory and he invited specialists from all over the world to the *Operator Theory Colloquium* at the Technische Universität Berlin.

Moreover, we consider him to be the creator of this series of *Workshops on Operator Theory in Krein Spaces*. Many of the participants of this workshop have experienced his friendship and his hospitality here in Berlin. It was the broard friendship to many of you - to most of the participants of this workshop which gave this workshop its special atmosphere. This friendship was a result of his life-long ties to so many of you. It was a result of his numerous visits to many of you and it was a result of his personality and his way of doing mathematics. It was his special mixture of profound and deep knowledge and his modest, calm and well-balanced attitude which made him the impressive personality he was. All of you know his silent but rigorous way of doing math, his uncompromising style of writing papers and his patient way of explaining mathematics to others.

In April 2007 Peter Jonas suddenly became seriously ill and after surgery and a short time of recovery he died on his 66th birthday on July 18th, 2007.

We will remember and miss him as a friend, colleague and teacher.

Jussi Behrndt, Karl-Heinz Förster, Carsten Trunk and Henrik Winkler

Local Perturbations on Absolutely Continuous Spectrum

V. Adamyan

In this talk we develop a local scattering theory for a finite spectral interval for pairs of self-adjoint operators, which are different extensions of the same densely definite symmetric operator. The obtained results are applied to the scattering problem for differential operators on graphs modeling real quantum networks of a quantum dot and attached semi-infinite quantum wires. We pay special attention to properties of obtained local scattering matrices in vicinities of resonances generated by eigenvalues of the energy operator for separated quantum dot.

Rigidity, Boundary Interpolation and Reproducing Kernels

D. Alpay

joint work with S. Reich and D. Shoikhet

Recall that a Schur function is a function analytic in the open unit disk and bounded by one in modulus there. When the angular convergence is replaced by the unrestricted one, the following rigidity result is due to D.M. Burns and S.G. Krantz, [2].

Theorem 1. Assume that a Schur function s satisfies

$$s(z) = z + O((1-z)^4), \quad z \hat{\to} 1,$$

where $\hat{\to}$ denotes angular convergence. Then $s(z) \equiv z$.

We use reproducing kernel methods, and in particular the results on boundary interpolation for generalized Schur functions proved in [1] to prove a general rigidity theorem which extend this result. The methods and setting allow us to consider the non-positive case. For instance we have the following result, which seems to be the first rigidity result proved for functions with poles.

Theorem 2. Let s be a generalized Schur function with one negative square and assume that

$$s(z) - \frac{1}{z} = O((1-z)^4), \quad z \hat{\to} 1.$$

Then $s(z) \equiv \frac{1}{z}$.

Details can be found in a manuscript on the arxiv site.

[1] D. Alpay, A. Dijksma, H. Langer, and G. Wanjala, Basic boundary interpolation for generalized Schur functions and factorization of rational J–unitary matrix functions, Operator Theory Advances Applications 165, 1–29, Birkhäuser, 2006.

[2] D.M. Burns and S.G. Krantz, Rigidity of holomorphic mappings and a new Schwarz lemma at the boundary, J. Amer. Math. Soc. 7 (1994), 661–676.

Iterates of the Schur Class Operator-Valued Function and Their Conservative Realizations

Yu. Arlinskiĭ

Let $\mathfrak{M}$ and $\mathfrak{N}$ be separable Hilbert spaces and let $\Theta(z)$ be the function from the Schur class $\mathbf{S}(\mathfrak{M},\mathfrak{N})$ of contractive functions holomorphic on the unit disk. The operator generalization of the Schur algorithm associates with Θ the sequence of contractions (the Schur parameters of Θ)

$$\Gamma_0 = \Theta(0) \in L(\mathfrak{M},\mathfrak{N}),\ \Gamma_n \in L(\mathfrak{D}_{\Gamma_{n-1}}, \mathfrak{D}_{\Gamma^*_{n-1}})$$

and the sequence of functions $\Theta_0 = \Theta$, $\Theta_n \in \mathbf{S}(\mathfrak{D}_{\Gamma_n}, \mathfrak{D}_{\Gamma^*_n})$, $n = 1,\ldots$ connected by the relations

$$\begin{aligned}\Gamma_n &= \Theta_n(0),\\ \Theta_n(z) &= \Gamma_n + zD_{\Gamma^*_n}\Theta_{n+1}(z)(I + z\Gamma^*_n\Theta_{n+1}(z))^{-1}D_{\Gamma_n},\ |z|<1.\end{aligned}$$

The function $\Theta_n(z)$ is called the n-th Schur iterate of Θ.

The function $\Theta(z) \in \mathbf{S}(\mathfrak{M},\mathfrak{N})$ can be realized as the transfer function $\Theta(z) = D + zC(I - zA)^{-1}B$ of a linear conservative and simple discrete-time system

$$\tau = \left\{\begin{bmatrix} D & C \\ B & A\end{bmatrix}; \mathfrak{M}, \mathfrak{N}, \mathfrak{H}\right\}$$

with the state space $\mathfrak{H}$ and the input and output spaces $\mathfrak{M}$ and $\mathfrak{N}$, respectively.

In this talk we give a construction of conservative and simple realizations of the Schur iterates Θ_n by means of the conservative and simple realization of Θ.

On the Ando-Khatskevich-Shulman Theorem

T.Ya. Azizov

A short proof of the Ando-Khatskevich-Shulman theorem about convexity and weak compactness of the image of the operator unit ball by a fractional linear transformation is given. We consider also an application of this result to the invariant subspace problem for non-contractive operators in Krein spaces.

This research was supported by the grant RFBR 05-01-00203-a of the Russian Foundation for Basic Researches.

Vector-Valued Logarithmic Residues and Non-Commutative Gelfand Theory

H. Bart

joint work with T. Ehrhardt and B. Silbermann

A vector-valued logarithmic residue is a contour integral of the type

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda)f(\lambda)^{-1}d\lambda \tag{1}$$

where D is a bounded Cauchy domain in the complex plane and f is an analytic Banach algebra valued function taking invertible values on the boundary ∂D of D. One of the main issues concerning such logarithmic residues is the following: if (1) vanishes, under what circumstances does it follow that f takes invertible values on all of D? A closely related question is: under what conditions does a Banach algebra have trivial zero sums of idempotents only? Recent developments to be discussed in the talk involve new aspects of non-commutative Gelfand theory.

Polynomial Stability: Some Recent Results and Open Problems

A. Batkai

We give a survey on the non-uniform asymptotic behaviour of linear opearator semigroups, concentrating on polynomial stability. The theory is applied to various concrete problems, such as hyperbolic systems or delay equations. Finally, a list of open problems will be presented.

Compact and Finite Rank Perturbations of Linear Relations

J. Behrndt

joint work with T.Ya. Azizov, P. Jonas and C. Trunk

For closed linear operators or relations A and B acting between Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$ the concepts of compact and finite rank perturbations can be defined with the help of the orthogonal projections P_A and P_B in $\mathcal{H}\oplus\mathcal{K}$ onto A and B. We discuss some equivalent characterizations for such perturbations and we show that these notions are natural generalizations of the usual concepts of compact and finite rank perturbations.

Two Parameter Eigencurves for Non-Definite Eigenproblems

P. Binding

A review will be given of some uses of the embedding

$$Ax = \lambda Bx - \mu x,$$

and in particular its (λ, μ) eigencurves, for studying the generalised eigenproblem

$$Ax = \lambda Bx.$$

Topics will include some history and properties of eigencurves; some classes of operators (from classical to recent) that they can accommodate; and some types of spectral questions that they can help to address.

Spectral Properties of the J-Self-Adjoint Operator Associated with the Periodic Heat Equation

M. Chugunova

joint work with D. Pelinovsky

The periodic heat equation has been derived as a model of the dynamics of a thin viscous fluid on the inside surface of a cylinder rotating around its axis. It is well known that the related Cauchy problem is generally ill-posed. We study the spectral properties of the J-self-adjoint operator associated with this equation. We will prove that this operator has compact inverse and does not have real eigenvalues. We shall also present numerical results. Some open questions will be stated.

Eigenvalue Problems with Boundary Conditions Depending Polynomially on the Eigenparameter

B. Ćurgus

Let S be a closed densely defined symmetric operator with equal defect numbers $d < \infty$ in a Hilbert space $(\mathcal{H}, \langle \cdot\,, \cdot \rangle)$. Let $\mathsf{b} : \mathrm{dom}(S^*) \to \mathbb{C}^{2d}$ be a boundary mapping for S. We assume that S has a self-adjoint extension with a compact resolvent. Let $\mathcal{P}(z)$ be a $d \times 2d$ matrix polynomial.

We will give sufficient conditions on $\mathcal{P}(z)$ under which the eigenvalue problem

$$S^* f = \lambda f, \quad \mathcal{P}(\lambda)\mathsf{b}(f) = 0$$

is equivalent to an eigenvalue problem for a self-adjoint operator $\widetilde{A}$ in a Pontrjagin space which is the direct sum of $\mathcal{H}$ and a finite-dimensional space. Both, this finite dimensional Pontrjagin space and the self-adjoint operator $\widetilde{A}$ are defined explicitly in terms of the coefficients of $\mathcal{P}(z)$.

In a special case when S is associated with an ordinary regular differential expression we give a description of the form domain of the operator $\widetilde{A}$ in terms of the essential boundary conditions. It is shown that the eigenfunction expansions for the elements in the form domain converge in a topology that is stronger than uniform.

If J is a self-adjoint involution on $\mathcal{H}$ and JS has a definitizable extension in the Krein space $(\mathcal{H}, \langle J\,\cdot\,, \cdot \rangle)$ our results extend to the eigenvalue problem in which S^* is replaced by JS^*.

As a model problem we propose the following

$$-f''(x) = \lambda (\operatorname{sgn} x) f(x), \quad x \in [-1, 1],$$

$$\begin{pmatrix} 1 & 0 & 0 & \lambda \\ 0 & 1 & -\lambda & \lambda^n \end{pmatrix} \begin{pmatrix} f(-1) \\ f(1) \\ f'(-1) \\ f'(1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

On Numbers of Negative Eigenvalues of some Products of Selfadjoint Operators

M.S. Denisov

Let $\mathcal{H}$ be a Hilbert space with a scalar product $(\cdot,\cdot)$. Let A and B be linear continuous selfadjoint operators with

$$\ker A = \{0\} \quad \text{and} \quad \ker B = \{0\}.$$

The main aim of this talk is to show the following: if

$$\sigma(A) \cap (-\infty, 0) \quad \big(\sigma(B) \cap (-\infty, 0)\big)$$

consist of m (n) negative eigenvalues counting the multiplicity then $\sigma(AB) \cap (-\infty, 0)$ and $\sigma(BA) \cap (-\infty, 0)$ contains at least $|n - m|$ eigenvalues.

The research was supported by the grant RFBR 05-01-00203-a of the Russian Foundation for Basic Researches.

A Jacobi Matrices Approach to Nevanlinna-Pick Problems

M. Derevyagin

joint work with A.S. Zhedanov

We propose a modification of the famous step-by-step process of solving the Nevanlinna-Pick problems for Nevanlinna functions. The process in question gives rise to three-term recurrence relations with coefficients depending on the spectral parameter. These relations can be rewritten in the matrix form by means of two Jacobi matrices. As a result of the considered approach, we prove a convergence theorem for multipoint Pade approximants to Nevanlinna functions.

On the Uniform Convergence of Pade Approximants for a Class of Definitizable Functions

V. Derkach

joint work with M. Derevyagin and P. Jonas

Let us say that a function ψ meromorphic in $\mathbb{C}_+$ belongs to the class $\mathbf{D}_{\kappa,-\infty}$ $(\kappa \in \mathbb{Z}_+)$ if $\psi(\lambda)/\lambda$ belongs to the generalized Nevanlinna class $\mathbf{N}_\kappa$ and for some $s_j \in \mathbb{R}$ $(j \in \mathbb{Z}_+)$ the following asymptotic expansion holds:

$$\psi(\lambda) = -\frac{s_0}{\lambda} - \frac{s_1}{\lambda^2} - \cdots - \frac{s_{2n}}{\lambda^{2n+1}} - \ldots \quad (\lambda \widehat{\rightarrow} \infty).$$

It is shown that for every $\psi \in \mathbf{D}_{\kappa,-\infty}$ there is a subsequence of diagonal Pade approximants, which converges to ψ locally uniformly on $\mathbb{C} \setminus \mathbb{R}$ in spherical metric. Conditions for the convergence of this subsequence on the real line are also found.

The Schur Transformation for Nevanlinna Functions: Operator Representations, Resolvent Matrices, and Orthogonal Polynomials

A. Dijksma

joint work with D. Alpay and H. Langer

We consider a fractional linear transformation for a Nevanlinna function n with a suitable asymptotic expansion at ∞, that is an analogue of the Schur transformation for contractive analytic functions in the unit disc. Applying the transformation p times we find a Nevanlinna function n_p which is a fractional linear transformation of the given function n. We discuss the effect of this transformation to the realizations of n and n_p, by which we mean their representations through resolvents of self-adjoint operators in Hilbert space.

Schwarz-Pick inequalities via transfer functions

M. Dritschel

joint work with M. Anderson and J. Rovnyak

We use unitary realizations to derive bounds on derivatives of arbitrary order for functions in the Schur-Agler class on the unit polydisk and ball.

The Riesz Basis Property of Indefinite Sturm-Liouville Problems with a Non Odd Weight Function

A. Fleige

For the Sturm-Liouville eigenvalue problem $-f'' = \lambda r f$ on $[-1, 1]$ with Dirichlet boundary conditions and with an indefinite weight function r changing it's sign at 0 we discuss the question whether the eigenfunctions form a Riesz basis of the Hilbert space $L^2_{|r|}[-1, 1]$. So far a number of sufficient conditions on r for the Riesz basis property are known. However, a sufficient and necessary condition is only known in the special case of an odd weight function r. We shall here give a generalization of this sufficient and necessary condition for certain generally non odd weight functions satisfying an additional assumption.

When are the Products of two Normal Operators Normal?

A. Gheondea

Given two normal operators A and B on a Hilbert space it is known that, in general, AB is not normal. Even more, I. Kaplansky had shown that it may be possible that AB is normal while BA is not. In this paper we address the question on (spectral) characterizations of those pairs of normal operators A and B for which both the products AB and BA are normal. This question has been solved for finite dimensional spaces by F.R. Gantmaher and M.G. Krein in 1930, and for compact normal operators A and B by N.A. Wiegmann in 1949. Actually, in these cases, the normality of AB is equivalent with that of BA. We consider the general case (no compactness assumption) by means of the Spectral Multiplicity Theorem for normal operators in the von Neumann's direct integral representation and the technique of integration/disintegration of Borel measures on metric spaces.

Unbounded Operators on Interpolation Spaces

K. Günther

Similar to the classical interpolation theory for bounded operators, we introduce - in general unbounded - operators S_0, S_1, S_Δ and S_Σ. If these operators are bounded, then we obtain the classical interpolation theory (see [Bergh, Löfström 1976]).

We investigate connections of the spectra of S_0, S_1, S_Δ and S_Σ and the spectra of the corresponding induced operators on interpolation spaces.

As an example, we consider ordinary differential operators on L^p-spaces.

Projective Hilbert Space Structures at Exceptional Points and Krein Space Related Boost Deformations of Bloch Spheres

U. Günther

joint work with B. Samsonov and I. Rotter

Simple non-Hermitian quantum mechanical matrix toy models are considered in the parameter space vicinity of Jordan-block structures of their Hamiltonians and corresponding exceptional points of their spectra. In the first part of the talk, the operator (matrix) perturbation schemes related to root-vector expansions and expansions in terms of eigenvectors for diagonal spectral decompositions are projectively unified and shown to live on different affine charts of a dimensionally extended projective Hilbert space. The monodromy properties (geometric or Berry phases) of the eigenvectors in the parameter space vicinities of spectral branch points (exceptional points) are briefly discussed.

In the second part of the talk, it is demonstrated that the recently proposed $\mathcal{PT}-$symmetric quantum brachistochrone solution [C. Bender et al, Phys. Rev. Lett. **98**, (2007), 040403, quant-ph/0609032] has its origin in a mapping artifact of the $\mathcal{PT}-$symmetric 2×2 matrix Hamiltonian in the vicinity of an exceptional point. Over the brachistochrone solution the mapping between the $\mathcal{PT}-$symmetric Hamiltonian as self-adjoint operator in a Krein space and its associated Hermitian Hamiltonian as self-adjoint operator in a Hilbert space becomes singular and yields the physical artifact of a vanishing passage time between orthogonal states. The geometrical aspects of this mapping are clarified with the help of a related hyperbolic Möbius transformation (contraction/dilation boost) of the Bloch (Riemann) sphere of the qubit eigenstates of the 2×2 matrix model.

The controversial discussion on the physics of the brachistochrone solution is briefly commented and a possible resolution of the apparent inconsistencies is sketched.

partially based on:

J. Phys. A 40 (2007) 8815-8833; arXiv:0704.1291 [math-ph].
arXiv:0709.0483 [quant-ph].

On Passive Discrete-Time Systems with a Normal Main Operator

S. Hassi

joint work with Yu. Arlinskiĭ and H. de Snoo

Linear discrete time-invariant systems τ are determined by the system of equations

$$\begin{cases} h_{k+1} = Ah_k + B\xi_k, \\ \sigma_k = Ch_k + D\xi_k, \end{cases} \qquad k = 0, 1, 2, \dots$$

where A, B, C, and D are bounded operators between the underlying separable Hilbert spaces $\mathfrak{H}, \mathfrak{M}$, and $\mathfrak{N}$. The system τ can be described by means of the block operator

$$T = \begin{pmatrix} D & C \\ B & A \end{pmatrix} : \begin{pmatrix} \mathfrak{M} \\ \mathfrak{H} \end{pmatrix} \to \begin{pmatrix} \mathfrak{N} \\ \mathfrak{H} \end{pmatrix}.$$

The system τ is said to be passive if T is contractive. In the talk the emphasis will be on systems whose main operator A is in addition normal. In particular, a general unitary similarity result for such systems is derived by means of a famous approximation result known for complex functions. The talk is a part of some joint work with Yury Arlinskiĭ and Henk de Snoo on so-called passive quasi-selfadjoint systems.

Hyponormal and Strongly Hyponormal Matrices in Inner Product Spaces

M.-A. Henn

joint work with C. Mehl and C. Trunk

The notions of hyponormal and strongly hyponormal matrices in inner product spaces with a possibly degenerate inner product are introduced. We study their properties and we give a characterization of such matrices. Moreover, we describe the connection to Moore-Penrose normal matrices and normal matrices.

Reconstruction of the Klein-Gordon Equation

R. Hryniv

We study the direct and inverse spectral problems related to the Klein–Gordon equations on $(0, 1)$,

$$-y''(x) + q(x)y(x) - (\lambda - p(x))2y(x) = 0,$$

that model a spinless particle moving in an electromagnetic field. Here $p(x) \in L^2(0, 1)$ and $q(x) \in W_2^{-1}(0, 1)$ are real-valued functions describing the electromagnetic field, and we impose suitable boundary conditions at the points $x = 0$ and $x = 1$. We give a complete description of possible spectra for such operators and solve the inverse problem of reconstructing p and q from the spectral data (two spectra or one spectrum and the corresponding norming constants).

Interpolation by Vector-Valued Analytic Functions with Applications to Controllability

B. Jacob

joint work with J.R. Partington and S. Pott

In this talk, norm estimates are obtained for the problem of minimal-norm tangential interpolation by vector-valued analytic functions, expressed in terms of the Carleson constants of related scalar measures. Applications are given to the controllability properties of linear semigroup systems with a Riesz basis of eigenvectors.

Forward-Backward Kinetic Equations and the Similarity Problem for Sturm-Liouville Operators

I. Karabash

Consider the equation

$$r(v)\psi_x(x,v) = \psi_{vv}(x,v) - q(v)\psi(x,v) + f(x,v),$$

$0 < x < 1$, $v \in \mathbb{R}$, and the associated half-range boundary value problem $\psi(0,v) = \varphi_+(v)$ if $v > 0$, $\psi(1,v) = \varphi_-(v)$ if $v < 0$. It is assumed that $vr(v) > 0$. So the weight function r changes its sign at 0. Boundary value problems of this type arise as various kinetic equations.

We consider the above equation in the abstract form

$$J\psi_x(x) + L\psi(x) = f(x),$$

where J and L are operators in a Hilbert space H such that $J = J^* = J^{-1}$, $L = L^* \geq 0$, and $\ker L = 0$. The case when L is nonnegative and has discrete spectrum or satisfies the weaker assumption $\inf \sigma_{ess}(L) > 0$ was described in great detail (see [4] and references therein). The latter assumption is not fulfilled for some physical models. The simplest example is the equation

$$v\psi_x(x,v) = \psi_{vv}(x,v), \quad 0 < x < 1, \quad v \in \mathbb{R}, \tag{1}$$

which was studied in a number of papers during last 50 years. The complete existence and uniqueness theory for equations of such type have not been constructed.

It will be shown that the method of [1] can be modified to prove the following theorem: *if the J-self-adjoint operator JL is similar to a self-adjoint one, then the associated half-range boundary problem has a unique solution for arbitrary $\varphi_\pm \in L^2(\mathbb{R}_\pm, |r|)$*. The latter can be applied to (1) due to the result of Fleige and Najman on the similarity of the operator $(\operatorname{sgn} v)|v|^{-\alpha}\frac{d^2}{dv^2}$, $\alpha > -1$. Connections between equations of type (1) and the recent papers [2,3] will be considered also.

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Complex Eigenvalues of Indefinite Sturm-Liouville Operators

Q. Katatbeh

Spectral properties of singular Sturm-Liouville operators of the form

$$A = \operatorname{sgn}(\cdot)\left(-\frac{d^2}{dx^2} + V\right)$$

with the indefinite weight $x \mapsto \operatorname{sgn}(x)$ on $\mathbb{R}$ are studied. For a class of potentials with $\lim_{|x|\to\infty} V(x) = 0$ the accumulation of complex and real eigenvalues of A to zero is investigated and explicit eigenvalue problems are solved numerically.

The KE-Problem: Description of Diagonal Elements

V. Khatskevich

joint work with V. Senderov

The authors continue their investigation. An affine f.l.m. $\mathcal{F}_A : \mathcal{K} \to \mathcal{K}$ of the unit operator-valued ball is considered in the case where the fixed point C of the continuation of $\mathcal{F}_A$ to $\overline{\mathcal{K}}$ is either an isometry or a coisometry. For the case in which one of the diagonal elements (for example, A_{11}) of the operator matrix A is identical, the structure of the other diagonal element (A_{22}) is studied completely. It is shown that, in all these reasonings, C cannot be replaced by an arbitrary point of the unit sphere; some special cases in which this is still possible are studied. In conclusion, the KE-property of the mapping $\mathcal{F}_A$ is proved.

The Similarity Problem for J-Nonnegative Sturm-Liouville Operators

A. Kostenko

joint work with I. Karabash and M. Malamud

We present new sufficient conditions for the similarity of J-self-adjoint Sturm-Liouville operators to self-adjoint ones. These conditions are formulated in terms of Weyl-Titchmarsh m-coefficients. This result is exploit to prove the regularity of the critical point zero for various classes of J-nonnegative Sturm-Liouville operators. In particular, we prove that 0 is a regular critical point of

$$A = (\operatorname{sgn} x)(-d^2/dx^2 + q(x))$$

if $q \in L^1(\mathbb{R}, (1+|x|)dx)$. Moreover, in this case A is similar to a self-adjoint operator if and only if it is J-nonnegative. We also show that the latter condition on q is sharp, that is we construct a potential $q_0 \in \cap_{\gamma<1} L^1(\mathbb{R}, (1+|x|^\gamma)dx)$ such that the operator A is J-nonnegative with the singular critical point zero and hence is not similar to a self-adjoint one.

Borg Type Uniqueness Theorems for Periodic Jacobi Operators with Matrix Valued Coefficients

A. Kutsenko

joint work with E. Korotyaev

We give a simple proof of Borg type uniqueness Theorems for periodic Jacobi operators with matrix valued coefficients.

Bissectors and Isometries on Hilbert Spaces

J.-P. Labrousse

Let $\mathcal{H}$ be a Hilbert space over $\mathbf{C}$ and let $F(\mathcal{H})$ be the set of all closed linear subspaces of $\mathcal{H}$. For all $M,\ N \in F(\mathcal{H})$ set $g(M, N) = \|P_M - P_N\|$ (known as the *gap metric*) where P_M, P_N denote respectively the orthogonal projections in $\mathcal{H}$ on M and on N.

For all $M,\ N \in F(\mathcal{H})$ such that

$$\ker(P_M + P_N - I) = \{0\},\ \Psi(M, N),$$

the *bissector* of M and N, is a uniquely determined element of $F(\mathcal{H})$ such that (setting $\Psi(M, N) = W$):

(i) $P_M P_W = P_W P_N$

(ii) $(P_M + P_N)P_W = P_W(P_M + P_N)$ is positive definite.

A mapping Φ of $F(\mathcal{H})$ into itself is called an *isometry* if

$$\forall M,\ N \in F(\mathcal{H}),\ g(M, N) = g(\Phi(M), \Phi(N)).$$

Theorem. Let $M,\ N \in F(\mathcal{H})$ such that $\ker(P_M+P_N-I) = \{0\}$ and let Φ be an isometry on $F(\mathcal{H})$. Then if $\ker(P_{\Phi(M)} + P_{\Phi(N)} - I) = \{0\}$:

$$\Phi(\Psi(M, N)) = \Psi(\Phi(M), \Phi(N)).$$

A number of applications of this result are given.

The Virozub–Matsaev Condition and Spectrum of Definite Type for Self-Adjoint Operator Functions

M. Langer

joint work with H. Langer, A. Markus and C. Tretter

The Virozub–Matsaev condition for self-adjoint operator functions and its relation to spectrum of positive type are discussed. The Virozub–Matsaev condition, e.g. implies the existence of a spectral function with nice properties. We consider, in particular, sufficient conditions and its connection with the numerical range.

Moment Problems for Real Measures on the Unit Circle

E. Lopushanskaya

joint work with M. Bakonyi

In this talk we are considering the following problem: when are the given complex numbers $(c_j)_{j=-n}^{n}$, $c_{-j} = \bar{c}_j$, the first moments of a real Borel measure $\mu = \mu^+ - \mu^-$ on $\mathbb{T}$, such that μ^- is supported on a set of at most k points. A necessary and sufficient condition is that the Toeplitz matrix $T = (c_{i-j})_{i,j=0}^{n}$ is a certain real linear combination of rank 1 Toeplitz matrices. For $k > 0$, this is more general than the condition that T admits self-adjoint Toeplitz extensions with k negative squares. For a singular T, an equivalent condition is that a certain polynomial has all its roots on $\mathbb{T}$. We also discuss the situation when T is invertible.

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More on the Operator Model for the Hydrogen Atom

A. Luger

joint work with P. Kurasov

The singular differential expression

$$\ell(y) := -y'' + \frac{q_0 + q_1 x}{x^2} y, \qquad x \in (0, \infty) \tag{1}$$

for $q_0 > \frac{3}{4}$ is in limit point case at the left endpoint 0, and hence the associated minimal operator is self-adjoint in $L_2(0, \infty)$.

In this talk we show a refinement of the earlier given model. More precisely, we introduce a Hilbert space $\mathcal{H}$ of (not necessarily square integrable) functions, in which a whole family of self-adjoint realizations of (1) is obtained by imposing certain (generalized) boundary conditions.

Finally we use these model operators in order to deduce a new expansion result.

Singular-Value-like Decompositions in Indefinite Inner Product Spaces

C. Mehl

The singular value decomposition is an important tool in Linear Algebra and Numerical Analysis. Besides providing a canonical form for a matrix A under unitary basis changes, it simultaneously displays the eigenvalues of the associated Hermitian matrices AA^* and A^*A. Similarly, one can ask the question if there is a canonical form for a complex matrix A that simlutaneously displays canonical forms for the complex symmetric matrices AA^T and A^TA.

In this talk, we answer this question in a more general setting involving indefinite inner products and defining an analogue of the singular-value decomposition in real or complex indefinite inner product spaces.

Structured Matrix Polynomials: Linearization and Condensed Forms

V. Mehrmann

joint work with R. Byers and H. Xu

We discuss general and structured matrix polynomials which may be singular and may have eigenvalues at infinity. We derive condensed/canonical forms that allow (partial) deflation of the infinite eigenvalue and singular structure of the matrix polynomial. The remaining reduced order staircase form leads to new types of linearizations which determine the finite eigenvalues and corresponding eigenvectors. The new linearizations also simplify the construction of structure preserving linearizations in the case of structures associated with indefinite scalar products.

On Trace Formula and Birman-Krein Formula for Pairs of Extensions

H. Neidhardt

joint work with J. Behrndt and M.M. Malamud

For scattering systems consisting of a (family of) maximal dissipative extension(s) and a selfadjoint extension of a symmetric operator with finite deficiency indices, the spectral shift function is expressed in terms of the abstract Titchmarsh-Weyl function. A variant of the Birman-Krein formula is proved.

Applications of Krein resolvent formula to localization on quantum graphs

K. Pankrashkin

joint work with F. Klopp

We study a special class of random interactions on quantum graphs, random coupling model. Using elementary facts from the theory of self-adjoint extensions we give some estimates for the spectral measures of such operators. This reduces the analysis of the localization problem to the well-known Aizenman-Molchanov method for discrete operators.

Inverse Spectral Problems for Sturm-Liouville Equation on Trees

V. Pivovarchik

joint work with R. Carlson

It turns out that well known Ambarzumian's theorem for Sturm-Liouville operator on an interval with Neumann boundary conditions at the endpoints can be generalized to the case of Sturm-Liouville operators on metric tree domains.

For the case of Dirichlet boundary conditions at the exterior vertices and continuity and Kirchhoff conditions at the interior vertex the inverse problem of recovering the potentials on the edges from the spectrum of the problem and the spectra of Dirichlet problems on the edges is solved for star shaped graphs.

The talk is based on results of [1], [2].

The work is supported by CRDF grant UK2-2811-OD-06.

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First Order Operators and Boundary Triples

O. Post

We introduce a first order approach to the abstract concept of boundary triples for Laplace operators. Our main application is the Laplace operator on a manifold with boundary; a case in which the ordinary concept of boundary triples does not apply directly. In our first order approach, we show that we can use the usual boundary operators also in the abstract Green's formula. Another motivation for the first order approach is to give an intrinsic definition of the Dirichlet-to-Neumann map and intrinsic norms on the corresponding boundary spaces. We also show how the first order boundary triples can be used to define a usual boundary triple leading to a Dirac operator.

On the Riesz Basis Property in Elliptic Eigenvalue Problems with an Indefinite Weight Function

S.G. Pyatkov

We consider elliptic eigenvalue problems with indefinite weight function of the form

$$Lu = \lambda Bu, \quad x \in G \subset R^n, \tag{1}$$

$$B_j u|_\Gamma = 0, \quad j = \overline{1, m}, \tag{2}$$

where L is an elliptic differential operator of order $2m$ defined in a domain $G \subset R^n$ with boundary Γ, the B_j's are differential operators defined on Γ, and $Bu = g(x)u$ with $g(x)$ a measurable function changing a sign in G. We assume that there exist open subsets G^+ and G^- of G such that $\mu(\overline{G^\pm} \setminus G^\pm) = 0$ (μ is the Lebesgue measure), $g(x) > 0$ almost everywhere in G^+, $g(x) < 0$ almost everywhere in G^-, and $g(x) = 0$ almost everywhere in $G^0 = G \setminus (\overline{G^+} \cup \overline{G^-})$. For example, it is possible that $G^0 = \emptyset$. Let the symbol $L_{2,g}(G \setminus G^0)$ stand for the space of functions $u(x)$ measurable in $G^+ \cup G^-$ and such that $u|g|^{1/2} \in L_2(G \setminus G^0)$. Define also the spaces $L_{2,g}(G^+)$ and $L_{2,g}(G^-)$ by analogy.

We study the problems on the Riesz basis property of eigenfunctions and associated functions of problem (1)-(2) in the weighted space $L_{2,g}(G \setminus G^0)$ and the question on unconditional basisness of "halves" of eigenfunctions and associated functions in $L_{2,g}(G^+)$ and $L_{2,g}(G^-)$, respectively. If $L > 0$ then these halves comprise eigenfunctions corresponding to positive and negative eigenvalues. The latter problem is closely related to the former. Our approach is based on the interpolation theory for weighted Sobolev spaces. We refine known results. Our conditions on the weight g are connected with some integral inequalities.

Inertia Theorems Based on Operator Lyapunov Equations

A. Ran

joint work with L. Lerer and I. Margulis

In 1962 D. Carlson and H. Schneider proved the following result. For an $n \times n$ complex matrix A let $\pi(A), \nu(A)$ and $\delta(A)$ denote the number of eigenvalues, counting multiplicities, located in the right halfplane, the left halfplane, and on the imaginary axis, respectively.

Theorem 1 *Let $A \in \mathbb{C}^{n\times n}$ and let X be a Hermitian matrix such that*

$$AX + XA^* = W \geq 0.$$

(i) *If $\delta(A) = 0$, then $\pi(X) \leq \pi(A)$ and $\nu(X) \leq \nu(A)$.*

(ii) *If X is nonsingular, then $\pi(A) \leq \pi(X)$ and $\nu(A) \leq \nu(X)$.*

(iii) *From (i) and (ii) it follows that if $\delta(A) = \delta(X) = 0$, then $\pi(X) = \pi(A)$ and $\nu(X) = \nu(A)$.*

The main goal of the lecture is to extend the third part of this result to the case of possibly unbounded linear operators acting on infinite dimensional Hilbert spaces. The second part was already generalized in earlier work of Curtain and Sasane. The following theorem is one of the main results.

Theorem 2 *Let $A : D(A) \subset \mathcal{H} \to \mathcal{H}$ be a linear, densely defined closed operator with domain $D(A)$. Suppose, $H \in L(\mathcal{H})$ is a self-adjoint invertible operator such that $\nu(H) < \infty$ and*

$$\langle (A^*H + HA)x, x\rangle \geq 0, \qquad \forall x \in D(A).$$

Assume, in addition, that A is boundedly invertible, the spectrum of A does not contain eigenvalues which lie on the imaginary axis, and $\sigma(A) \cap \mathbb{C}^-$ is a bounded spectral set. Then

$$\nu(H) = \nu(A).$$

The proof of the theorem makes use of the theory of operators in indefinite inner product spaces.

Canonical Structures for Palindromic Matrix Polynomials

L. Rodman

joint work with P. Lancaster and U. Prells

We study spectral properties and canonical structures of palindromic matrix polynomials in terms of their linearizations, standard triples, and unitary triples. These triples describe matrix polynomials via eigenvalues and Jordan chains. As an application of canonical structures and their properties, we develop criteria for stable boundedness of solutions of systems of linear differential equations with symmetries. Open problems will be mentioned.

Passive Impedance Bi-Stable Systems with Losses of Scattering Channels

N.A. Rozhenko

joint work with D.Z. Arov

The conservative and passive impedance linear time invariant systems $\Sigma = (A, B, C,\ D; X, U)$ with discrete time and with Hilbert state and external spaces X and U respectively, and their impedances $c(z) = D + zC(I - zA)^{-1}B$ were studied earlier by different authors, see e.g. [1]. In our recent works [3]-[5] we concentrate our attention on the losses case. By this we understand the case, when the factorization inequalities

$$\varphi(z)^*\varphi(z) \leq 2\Re c(z), \quad \psi(z)\psi(z)^* \leq 2\Re c(z), \quad z \in D,$$

have at least one nonzero solution $\varphi(z)$ and $\psi(z)$ in the classes of holomorphic inside open unite disk D functions with values from $\mathbb{B}(U, Y_\varphi)$ and $\mathbb{B}(U_\psi, U)$, respectively. Moreover, main results are relate to the bi-stable systems , i.e. to such systems, in which main operator A is a contraction from the class C_{00} that means

$$A^n \to 0 \quad \text{and} \quad (A^*)^n \to 0 \quad \text{when } n \to \infty.$$

To the impedance system Σ of such type corresponds the passive impedance systems with losses for which even factorizations equation

$$\varphi(\zeta)^*\varphi(\zeta) = 2\Re c(\zeta), \quad \psi(\zeta)\psi(\zeta)^* = 2\Re c(\zeta), \quad \text{a.e. } |\zeta| = 1,$$

have nonzero solutions, which understands in weak sense for operator-valued functions. Such a system with impedance matrix $c(z)$ can be realized as a part of scattering-impedance lossless transmission minimal system $\tilde{\Sigma} = (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}; \tilde{X}, \tilde{U}, \tilde{Y})$ with $\tilde{U} = U_1 \oplus U \oplus U_2$, $\tilde{Y} = Y_1 \oplus U \oplus Y_2$, where $U_2 = Y_2 = U$, by setting

$$X=\tilde{X},\ A=\tilde{A},\ B=\tilde{B}|U,\ C=P_U\tilde{C} \text{ and } D=P_U\tilde{D}|U.$$

The system $\tilde{\Sigma}$ has the system operator

$$M_{\tilde{\Sigma}}=\begin{bmatrix} A & K & B & 0\\ M & S & N & 0\\ C & L & D & I_U\\ 0 & 0 & I_U & 0 \end{bmatrix}$$

that is $(\tilde{J}_1,\tilde{J}_2)$-unitary; transfer function $\tilde{\theta}_{J_1,J_2}(z)$ of system $\tilde{\Sigma}$ when $z\in D$ is (J_1,J_2)-bi-inner (in a certain weak sense) and has special structure

$$\tilde{\theta}_{J_1,J_2}(z)=\begin{bmatrix} \alpha(z) & \beta(z) & 0\\ \gamma(z) & \delta(z) & I_U\\ 0 & I_U & 0 \end{bmatrix}, \qquad z\in D,$$

with 22-block $\delta(z)$ that equal to the impedance matrix $c(z)$ that belongs to the Caratheodory class, where

$$J_1=\begin{bmatrix} I_{U_1} & 0\\ 0 & J_U \end{bmatrix}, J_2=\begin{bmatrix} I_{Y_1} & 0\\ 0 & J_U \end{bmatrix},$$

$$J_U=\begin{bmatrix} 0 & -I_U\\ -I_U & 0 \end{bmatrix}, \tilde{J}_j=\begin{bmatrix} I_X & 0\\ 0 & J_j \end{bmatrix},$$

$j=1,2$. If the main operator A of the system Σ belongs to the class C_0 in Nagy-Foias sense, then function $\tilde{\theta}_{J_1,J_2}(z)$ is meromorphic in the exterior D_e of disk D with bounded Nevanlinna characteristic in D_e. Moreover, meromorphic pseudocontinuation in D_e in weak sense of the restriction of $\tilde{\theta}_{J_1,J_2}(z)$ on D equals to $\tilde{\theta}_{J_1,J_2}(z)$ in D such that for any $\tilde{u}\in\tilde{U},\tilde{y}\in\tilde{Y}$

$$\lim_{r\downarrow 1}(\tilde{\theta}_{J_1,J_2}(r\zeta)\tilde{u},\tilde{y})=\lim_{r\uparrow 1}(\tilde{\theta}_{J_1,J_2}(r\zeta)\tilde{u},\tilde{y}) \qquad \text{a.e. } |\zeta|=1.$$

Impedance matrix $c(z)$ of system Σ as a block of $\tilde{\theta}_{J_1,J_2}(z)$ is meromorphic in D_e with bounded Nevanlinna characteristic in D_e and for any $u_1,u_2\in U$

$$\lim_{r\downarrow 1}(c(r\zeta)u_1,u_2)=\lim_{r\uparrow 1}(c(r\zeta)u_1,u_2) \qquad \text{a.e. } |\zeta|=1.$$

Our results are intimately connected with work [2] where problems related to Surface Acoustic Wave filters are studied. In corresponding systems inputs are incoming waves and voltages and outputs are outgoing waves and currents, and transfer function is "mixing matrix"

$$\begin{bmatrix} \alpha & \beta\\ \gamma & \delta \end{bmatrix}$$

that is the main part of transmission matrix $\tilde{\theta}_{J_1,J_2}$ of system $\tilde{\Sigma}$ in our considerations.

In the case $\dim U<\infty$ the analytical problem of the description of the set of corresponding lossless scattering-impedance transmission matrices with given 22-block $\delta=c$ was studied in [3]. The present here results can be found in [4], [5].

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J-Theory and Random Matrices

L.A. Sakhnovich

We consider a special case of Riemann-Hilbert problem, which can be formulated in terms of J-theory. The given matrix R(x) is J- module. We investigate the connections of this Riemann-Hilbert problem with the canonical differential systems and random matrix theory. The asymptotic formulas are deduced.

Invariant Nonnegative Relations in Hilbert Spaces

A. Sandovici

The concept of μ-scale invariant operator with respect to a unitary transformation in a separable Hilbert space is extended to the case of linear relations (multi-valued linear operators). It is shown that if S is a nonnegative linear relation which is μ-scale invariant for some $\mu > 0$, then its adjoint S^* and its extremal nonnegative selfadjoint extensions are also μ-scale invariant.

Asymptotics of Eigenvalues of a Sturm–Liouville Problem with Discrete Self-Similar Indefinite Weight

I.A. Sheipak

joint work with A. A. Vladimirov

We study the asymptotics of the spectrum for the boundary eigenvalue problem

$$-y'' - \lambda\rho y = 0, \tag{1}$$

$$y(0) = y(1) = 0, \tag{2}$$

where $\rho \in \overset{\circ}{W}{}_2^{-1}[0,1]$ is the generalized derivative of fractal (self-similar) piece-wise function $P \in L_2[0,1]$.

The numbers $n \in \mathbb{N}$, $n \geqslant 2$, q ($|q| > 1$), $Z_+, Z_- \geqslant 0$ can be defined via parameters of self-similarity of function P.

Our main results are the following:

1. If $q > 1$, $Z_+ > 0$ and $Z_+ + Z_- = n-1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \dots, Z_+$, such that positive eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{l+kZ_+} = \mu_l \cdot q^k(1 + o(1)).$$
2. If $q > 1$, $Z_- > 0$ and $Z_+ + Z_- = n-1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \dots, Z_-$, such that negative eigenvalues $\{\lambda_{-k}\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{-(l+kZ_-)} = -\mu_l \cdot q^k(1 + o(1)).$$
3. If $q < -1$, $Z_+ + Z_- = n - 1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \dots, n-1$, such that that positive eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{l+k(n-1)} = \mu_l \cdot |q|^{2k}(1 + o(1))$$
and negative eigenvalues $\{\lambda_{-k}\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{-(l+Z_-+k(n-1))} = -\mu_l \cdot |q|^{2k+1}(1 + o(1)).$$

All these results are new even if function ρ is positive.

Asymptotics of eigenvalues of Sturm–Liouville problem with discrete self-similar weight (http://arxiv.org/abs/0709.0424)

Dissipative Operators in Krein Space. Invariant Subspaces and Properties of Restrictions

A.A. Shkalikov

We prove that a maximal dissipative operator in Krein space has a maximal nonnegative invariant subspace provided that the operator admits a matrix representation and the upper right operator in this representation is compact relative to the lower right operator. Under weaker assumptions this result was obtained (in increasing order of generality) by Pontrjagin, Krein, Langer and Azizov.

The main novelty is that we start the investigation of properties of the restrictions onto invariant subspaces. In particular, we find sufficient conditions for the restrictions to be generators of holomorphic or C_0- semigroups.

On Realizations of Supersymmetric Dirac Operator with Aharonov - Bohm Magnetic Field

Yu. Shondin

We consider operator models for the supersymmetric Dirac Hamiltonian (supercharge) H describing electron moving in the singular Aharonov-Bohm magnetic field. In the standard model of H one takes a special self-adjoint extension H^s of the minimal operator which is uniquely determined as the self-adjoint extension of the minimal operator satisfying the conditions: 1) H^s is supersymmetric; 2) with $(H^s)^2 = \text{ diag }(H^+, H^-)$ it holds $H^+ \geq H^-$ if $\nu > 0$ and $H^+ \leq H^-$ if $\nu < 0$. It occurs that the spectral shift functions $\xi(\lambda)$ for the pair $\{H^+, H^-\}$ associated with H^s is equal to $\xi(\lambda) = (\nu - [\nu])\theta(\lambda)$. However, this differs from the case of regular magnetic field with the same value of magnetic flux, where the corresponding spectral shift function $\xi(\lambda) = \nu\theta(\lambda)$. This difference comes from the presence of $[|\nu|]$ zero modes in the regular case.

We compare this with proposed nonstandard models of H based on the extension theory in Pontryagin spaces. Particularly, we discuss $[|\nu|]$-parametric family of realizations of the Dirac and Pauli Hamiltonians in Pontryagin spaces which have the same number of zero modes as in the case of regular magnetic field with the same value of magnetic flux.

On Spectralizable Operators

V. Strauss

joint work with C. Trunk

We introduce the notion of spectralizable operators. A closed operator A in a Hilbert space spectralizable if there exists a non-constant polynomial p such that the operator $p(A)$ is a spectral operator in the sense of Dunford. We show that such operators belongs to the class of generalized spectral operators and give some examples where spectralizable operators occur naturally.

On Reducing of Selfajoint Operators to Diagonal Form

L. Sukhocheva

We shall discuss the infinite dimensional analog of the following well-known result:

Let A and B be selfadjoint $n \times n$ matrices and let B be non-degenerate. Then the pair A and B can be reduced to the diagonal form if one of the following assumptions holds.

(i) matrix $B^{-1}A$ is similar to a selfajoint one;

(ii) there exists a positive matrix S^{-1} such that $S^{-1}A$ and $S^{-1}B$ commute.

The research was supported by the grant RFBR 05-01-00203-a of the Russian Foundation for Basic Researches.

A Look at Krein Space: New Thoughts and Old Truths

F.H. Szafraniec

I am going to resume the theme of my talk given at 5th Workshop. As that, presented by 'chalk & blackboard' means, was badly organized this time I would like to hope the beamer presentation to help me in achieving the goal. Anyway, among the topics I intend to place in there are: a proposal for generalizing the notion of Krein space and a way to unify different kind of extensions.

On Spectrum of Quantum Dot with Impurity in Lobachevsky Plane

M. Tušek

A model of the quantum dot with impurity in Lobachevsky plane is considered. With explicit formulae for the Green function and the Krein $Q-$function in hand, a numerical analysis of the spectrum is done. The analysis turns out to be more complicated than one might expect at first glance since spheroidal functions with general characteristic exponent are involved. The curvature effect on the eigenvalues and the eigenfunctions is investigated.

Perturbation Bounds for Relativistic Spectra

K. Veselić

The newly developed perturbation theory for finite eigenvalues is applied to typical cases with spectral gaps: (i) the Dirac operator with the Coulomb potential and (ii) the supersymmetric Dirac oscillator. Sharp relative bounds are obtained.

Products of Nevanlinna Functions with Certain Rational Functions

R. Wietsma

Let Q be a scalar Nevanlinna function and let r be a rational function with real poles and zeros, which is also real on the real axis. Then the product $\widetilde{Q} = rQ$ (with special choices of r) is considered. In particular, the operator representation of the function $\widetilde{Q}$ is connected to the operator representation of the function Q. Furthermore, the connections between the models of the functions Q and $\widetilde{Q}$ are studied, involving the $L^2(\mathrm{d}\sigma)$-models and RKS-models.

Commuting Domination in Pontryagin Spaces

M. Wojtylak

We will discuss the following theorem, proved originally in [2] for formally normal and normal operators in Hilbert spaces.

Theorem. Let $A_0, \ldots, A_n$ $(n \geq 1)$ be symmetric operators in a Pontryagin space $\mathcal{K}$ and let $\mathcal{E}_{ij}$, $0 \leq i < j \leq n$, be dense linear spaces of $\mathcal{K}$ such that

(i) A_j weakly commutes with A_0 on $\mathcal{E}_{0j}$ for $j = 1, \ldots, n$;

(ii) A_i pointwise commutes with A_j on $\mathcal{E}_{ij}$ for $1 \leq i < j \leq n$;

(iii) A_0 is essentially selfajoint on $\mathcal{E}_{0j}$ for $j = 1, \ldots, n$;

(iv) A_0 dominates A_j on $\mathcal{E}_{0j}$ for $j = 1, \ldots, n$.

Then $\bar{A}_0, \ldots, \bar{A}_n$ are spectrally commuting selfadjoint operators.

The proof requires some results on bouded vectors of a selfadjoint operator in a Pontryagin space. As a corollary we obtain a polynomial version of Nelson's criteria for selfadjointness. We will use the theory of operator matrices with all unbounded entries, which was developed in [1].

[1] M. Möller, F.H. Szafraniec, Adjoints and formal adjoints of matrices of unbounded operators, *Proc. Amer. Math. Soc.*

[2] J. Stochel, F. H. Szafraniec, Domination of unbounded operators and commutativity, *J. Math. Soc. Japan,* 55 No.2, (2003), 405-437.

[3] M. Wojtylak, Commuting domination in Pontryagin spaces, preprint.