A Cauchy problem associated with an (ω, W)-dissipative operator

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Let A be a closed densely defined operator in a Hilbert space $\{\mathcal{H}, (\,\cdot\,,\,\cdot\,)\}$. Let W be a bounded self-adjoint operator on $\mathcal{H}$ with $0 \in \rho(W)$, the resolvent set of W, and let ω be a real number. The operator A will be called (ω,W)-dissipative if

$$\mathrm{Re}\,(WAx, x) \leq \omega(x, x), \quad x \in \mathrm{dom}\, A.$$

Our main result is contained in the following theorem.

Theorem. Let A be a closed densely defined and maximal (ω, W)-dissipative operator on a Hilbert space $\{\mathcal{H}, (\,\cdot\,,\,\cdot)\}$, where $\omega \in \mathbf{R}$ and W is a bounded self-adjoint operator on $\mathcal{H}$ with $0 \in \rho(W)$. If dom A contains a maximal uniformly negative subspace of the Krein space $\{\mathcal{H}, [\,\cdot\,,\cdot\,]_W = (W\cdot, \cdot)\}$, then the Cauchy problem

$$\begin{cases} x'(t) &= A\,x(t), \quad t \in [0, \infty), \\ x(0) &= x_0 \in \mathrm{dom}\, A \end{cases}$$

is uniformly correct.

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