Aymptotic Expression for Non-stationary Perturbation Determinants

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Let A be a selfadjoint operator in the Hilbert space $\mathcal{H}_1$, and

$$H = \begin{pmatrix} A+V & B \\ B^* & D \end{pmatrix}, \; H_0 = \begin{pmatrix} A+V & 0 \\ 0 & D \end{pmatrix}, \tag{1}$$

be self-adjoint operators in the Hilbert space $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$. Assuming that V and B are nuclear and, additionally, that the spectra of $A+V$ and D are weakly separated, that is, for some $\alpha \in \mathbb{R}$,

$$\max \sigma(A+V) \leq \alpha \leq \min \sigma(D) \tag{2}$$

and α is not an eigenvalue of $A+V$ and of D we consider the *partial non–stationary perturbation determinant* defined by the relation

$$\Delta_1(t) := \det\left(e^{itA} P_1 e^{-itH}|_{\mathcal{H}_1}\right), \qquad t > 0,$$

where P_1 is the orthogonal projection in $\mathcal{H}$ onto $\mathcal{H}_1$. If $V \leq 0$ and the spectrum of H is absolutely continuous in at least one of the intervals $(-\infty, \alpha]$, $[\alpha, +\infty)$ then

$$\Delta_1(t) \underset{t \to \infty}{=} e^{-b-iat}\left(1 + o(1)\right)$$

with

$$a = \int_{-\infty}^{\alpha} \xi_{H/H_0}(\lambda)\, d\lambda + \operatorname{tr} V \geq 0,$$

where $\xi_{H/H_0}(\lambda)$ is the spectral shift function for the pair H, H_0, and $b \geq 0$, and $a = 0$ or $b = 0$ if and only if $B = 0$. If $V = 0$ and the block B is finite–dimensional, then

$$\Delta_1(t) = \exp\left(-\int_0^t \operatorname{tr}\left(\Gamma^s(s,s)\right)\, ds\right).$$

where $\Gamma^t(s,s')$, $0 \leq s, s' \leq t$, is the matrix Fredholm resolvent kernel of the defined by A and B system of second kind Fredholm integral equations on $(0,t)$.

Under additional assumptions the above assertions are true also for J -sefadjoint operators with

$$J = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

The talk reported on is based on results of a joint work with Heinz Langer.