On Hamiltonian operators and operator-functions

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This talk is based on a joint work with A. Dijksma and I.V. Gridneva.

Let $\{\mathcal{G}, (\cdot\,,\cdot\,)\}$ be a Hilbert space. Consider the orthogonal direct sum

$$\mathcal{H} = \mathcal{G} \oplus \mathcal{G},$$

which is a Hilbert space whose inner product we also denote by $(\cdot\,,\cdot\,)$. Introduce the matrices

$$J = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

and

$$\mathfrak{J} = \begin{bmatrix} 0 & iI \\ -iI & 0 \end{bmatrix}$$

and two Krein spaces $\mathcal{K}_J := \{\mathcal{H}, [\cdot\,,\cdot\,]_J\}$ and $\mathcal{K}_{\mathfrak{J}} := \{\mathcal{H}, [\cdot\,,\cdot\,]_{\mathfrak{J}}\}$, whose indefinite inner products are defined by

$$[\cdot\,,\cdot\,]_J = (J\cdot\,,\cdot\,) \text{ and } [\cdot\,,\cdot\,]_{\mathfrak{J}} = (\mathfrak{J}\cdot\,,\cdot\,),$$

respectively.

A closed densely defined operator $\mathfrak{A}$ on $\mathcal{H}$ is called a Hamiltonian operator if $i\mathfrak{A}$ is self-adjoint in the Krein space $\mathcal{K}_{\mathfrak{J}}$ and $\mathfrak{A}$ is called a nonnegative Hamiltonian operator if it is a Hamiltonian operator and $i\mathfrak{A}$ is dissipative in the Krein space $\mathcal{K}_J$. The aim of this talk is to give conditions which imply the boundedness of closed densely defined Hamiltonian operators and conditions by which a Hamiltonian operator-function is reducible.

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