Sturm-Liouville Problems with Indefinite Weights

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In this talk I will consider a regular indefinite Sturm-Liouville eigenvalue problem

$$-(pf')' + qf = \lambda\, r\, f \quad \text{on} \quad [-1,1],$$

subject to general self-adjoint boundary conditions. Here $1/p, q, r$, are real integrable functions on $[-1,1]$ and $p(x) > 0$ and $x\, r(x) > 0$ on $[-1,1]$. The spectrum of this problem is discrete. The eigenfunctions and the generalized eigenfunctions form a complete set in the Hilbert space $L_{2,|r|}(-1,1)$. I will discuss the following question: Does there exist a Riesz basis of $L_{2,|r|}(-1,1)$ which consists of eigenfunctions and generalized eigenfunctions of the problem? In particular I will consider how the answer to this question depends on the boundary conditions.