Spectral properties of some class of operator matrices

Rostyslav Hryniv

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Let X, Y, Z be Banach spaces. With linear (in general unbounded) operators

$$A \text{ in } X, \quad D \text{ in } Y, \quad C \text{ from } X \text{ into } Y, \quad B \text{ from } Y \text{ into } X,$$

$$\Gamma_X \text{ from } X \text{ into } Z, \quad \Gamma_Y \text{ from } Y \text{ into } Z,$$

we associate an operator matrix

$$\mathcal{A} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

acting in the space $\mathcal{X} := X \times Y$ on the domain

$$\mathcal{D}(\mathcal{A}) = \{(x, y) \in \mathcal{X} \mid x \in \mathcal{D}(A) \cap \mathcal{D}(C), y \in \mathcal{D}(B) \cap \mathcal{D}(D), \Gamma_X x = \Gamma_Y y\}.$$

Typical examples of problems that give rise to the operator $\mathcal{A}$ include, e.g.,

(i) λ-rational Sturm–Liouville problems;

(ii) dynamical problems describing the motion of a Markovian particle that moves in a domain Ω according to a diffusion law and interacts nontrivially with the boundary $\partial\Omega$;

(iii) differential equations with delay in a Banach space.

Under suitable assumptions on the operators involved, we study the question of closability of $\mathcal{A}$ and describe its closure, determine the essential spectrum, and discuss generation of the analytic semigroup.