Spectrum of definite type of self-adjoint operators in Krein spaces

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For a self-adjoint operator $\mathcal{A}$ in a Krein space $\mathcal{K}$ we construct an interval $[\nu, \mu]$ outside of which the operator has only spectrum of definite type and possesses a local spectral function. The numbers μ and ν are constructed by means of spectral properties of the operators A, B and D, where

$$\mathcal{A} = \begin{bmatrix} A & B \\ -B^* & D \end{bmatrix}$$

is the block operator matrix representation of $\mathcal{A}$ with respect to an arbitrary fundamental decomposition of $\mathcal{K}$.

As a consequence, a spectral subspace $\mathcal{L}_\Delta(\mathcal{A})$ corresponding to an interval Δ outside $[\nu, \mu]$ admits an angular operator representation; if, e.g., Δ is of positive type, this means

$$\mathcal{L}_\Delta(\mathcal{A}) = \left\{ \begin{pmatrix} x \\ K_1^\Delta x \end{pmatrix} : x \in \mathcal{H}_1^\Delta \right\}.$$

We describe a defect subspace of the domain $\mathcal{H}_1^\Delta$ of the angular operator in terms of the Schur complement of $\mathcal{A}$.