DIFFERENT BOUNDARY VALUE PROBLEMS ASSOCIATED WITH A HIGH ORDER SINGULAR PERTURBATION

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In this lecture we report on joint work with A.Dijksma and P.Kurasov. Let $\mathcal{H}$, $\langle\cdot,\cdot\rangle$ be a Hilbert space, L be a positive self-adjoint operator in $\mathcal{H}$ and φ be an element of the space $\mathcal{H}_{-n}\setminus\mathcal{H}_{-n+1}$, $n\geq 3$. Here $\mathcal{H}_m$'s are the Hilbert scale spaces with the inner products $\langle (L+1)^m\cdot,\cdot\rangle$. Also we choose $n-1$ positive real points $a_1,a_2,\ldots,a_{n-1}$ and associate with them the polynomials $b_0(z)=1$ and $b_j(z)=(z+a_1)(z+a_2)\cdots(z+a_j)$, $j=1,\ldots,n-1$.

With the L and φ we associate two suitable inner product spaces and two maximal operators whose domains contain the the elements $\frac{1}{L-z}\varphi,\quad z\in\rho(L)$. This gives rise to two different kinds of self-adjoint realizations (two models A and B) of the formal singular perturbation $L_\theta = L+\operatorname{tg}\theta\,\langle\,\cdot\,,\varphi\rangle\,\varphi$ and we explain what the models have in common and where they differ. More precisely the B-model describes minimal realization of the generalized Nevanlinna function

$$Q_B(z)=b_{n-1}(z)\langle\frac{1}{L-z}\varphi,\frac{1}{b_{n-1}(L)}\varphi\rangle+p_{n-2}(z),$$

where $p_{n-2}(z)=c_0+c_1b_1(z)+\cdots+c_{n-2}b_{n-2}(z)$ is a polynomial with real coefficients. The function $Q_B(z)\in N_\kappa$ with $\kappa=[\frac{n-1}{2}]$ and admits representation

$$Q_B(z)=b_{n-2}(z)Q_A(z),\ Q_A(z)=(z+a_{n-1})\langle\frac{1}{L-z}\varphi,\frac{1}{b_{n-1}(L)}\varphi\rangle+\frac{p_{n-2}(z)}{b_{n-2}(z)}.$$

The function $Q_A(z)$ belongs to the class $N_{\kappa'}$ with $0\leq\kappa'\leq n-2$ negative squares and the A-model describes a minimal realization of this function. The generalized Nevanlinna function Q_A (Q_B) is a Q-function of a symmetric operator $A_{\min}$ ($B_{\min}$) and a self-adjoint extension A_0 (B_0) in a Pontryagin space $\mathcal{H}_A$ ($\mathcal{H}_B$) correspondingly. The one-parameter family of self-adjoint extensions of $A_{\min}$ ($B_{\min}$) is interpreted as the family of realizations $A_\theta(B_\theta)$ of L_θ in the $A(B)$-model correspondingly.

Considering the realizations A_θ and B_θ as suitable restrictions of the corresponding maximal operators $A^*_{\min}$ and $B^*_{\min}$ we get different boundary value problems associated with L_θ. From their comparison we obtain a complete correspondence between models A and B.