On Intervals of Type π

Carsten Trunk

This talk is based on a joint work with T. Ya. Azizov and P. Jonas.

Let $A = A^+$ be a selfadjoint operator in a Krein space $(\mathcal{H}, [\cdot,\cdot])$ with $\rho(A) \neq \emptyset$. In [1] the notion of points of plus type was introduced. A point λ from the approximative point spectrum $\sigma_{ap}(A)$ of A is called of *plus type* if there exist numbers $\epsilon > 0$, $\delta > 0$ such that

$$x \in \mathcal{D}(A),\ \|x\| = 1,\ \|(A-\lambda)x\| \leq \epsilon \quad \Rightarrow \quad [x,x] \geq \delta.$$

In this talk we call a point λ, $\lambda \in \sigma_{ap}(A)$, a point of *type* π_+ if there exist a subspace $\mathcal{H}_0 \subset \mathcal{H}$ with codim $\mathcal{H}_0 < \infty$ and numbers $\epsilon > 0$, $\delta > 0$ such that

$$x \in \mathcal{H}_0 \cap \mathcal{D}(A),\ \|x\| = 1,\ \|(A-\lambda)x\| \leq \epsilon \quad \Rightarrow \quad [x,x] \geq \delta.$$

Let $[a,b]$ be an interval such that each point of $[a,b]$ is either a point of type π_+ or a point from $\rho(A)$. We show that there are at most finitely many points of $[a,b]$ which are not of plus type. Under an additional assumption no point of $[a,b]$ is an accumulation point of the nonreal spectrum of A and we give an estimate for the growth of the resolvent near $[a,b]$, that is A is locally definitizable over a neighbourhood of $[a,b]$.

Moreover, let B be a selfadjoint operator $(\mathcal{H}, [\cdot,\cdot])$ which arises from A by a compact perturbation (in resolvent sense) and let λ_0 be a point of type π_+ with respect to A. Then it can be shown that λ_0 is of type π_+ with respect to B.

[1] Lancaster, P., Markus, A.S., and Matsaev, V.I.: Definitizable Operators and Quasihyperbolic Operator Polynomials, J. Funct. Anal. **131** (1995), 1–28