An indefinite analogue of the Stieltjes class

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The Stieltjes class (S) is the set of all functions analytic on $\mathbb{C} \setminus [0, +\infty)$ with $\operatorname{Im} f(z) \geq 0$ for $\operatorname{Im} z > 0$ and $f(x) \geq 0$ for $x \in (-\infty, 0)$. This class plays an important role in the study of the vibrations of a string with inhomogenuous mass distribution. We introduce and investigate an indefinite analogue of the Stieltjes class. In fact we show that the class $\mathcal{N}_{<\infty}^{ep}$ which contains those generalized Nevanlinna functions which are analytic on $\mathbb{C} \setminus [0, +\infty)$ with possible exception of finitely many poles is a proper indefinite analogue of (S).