# Some Class of Solvable Potentials for Dirac Type Systems

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Let $\omega_{\pm}(\lambda)$, $-\infty < \lambda < \infty$, be a function of limited variation such that

$$\sigma(\lambda) := \frac{\lambda}{2\pi} + \omega_{+}(\lambda),$$

is non-decreasing and $\omega_{-}(\lambda)$ is a non-decreasing step function having $\kappa < \infty$ jump discontinuities. Set

$$H(t) = H_{+}(t) - H_{-}(t),\ H_{\pm}(t) = \int\limits_{-\infty}^{\infty} e^{-i\lambda t} d\left[\omega_{+}(\lambda) - \omega_{-}(\lambda)\right],\ -\infty < t < \infty.$$

Then for any positive $r < \infty$ the integral operator $\widehat{H}_r = \widehat{H}_{+,r} - \widehat{H}_{-,r}$ in $\mathbb{L}^2(0,r)$ with the difference kernel $H(t-s)$ is nuclear and there is $r_0 > 0$ such that for $r \geq r_0$ the operator $I + \widehat{H}_r$ is invertible and has exactly $\kappa$ negative eigenvalues. For any $r > 0$ such that $I+ \widehat{H}_r$ is invertible let $\Gamma_r(t,s), 0 \leq t, s \leq r$, $r \geq r_0$, be the unique continuous solution of the integral equation

$$\Gamma_r(t,s) + \int\limits_0^r H(t-u)\Gamma_r(u,s)du = H(t-s).$$

We consider the introduced by M.G. Krein continual analog of the system of orthogonal trigonometric polynomials on the unit circle:

$$e(r,\lambda) := e^{i\lambda r}\left(1 - \int\limits_0^r \Gamma_r(s,r)e^{-i\lambda s}ds\right),\ 0 \leq r < \infty,$$

$$\left(\int_{-\infty}^{\infty} e(r,\lambda)^* e(r',\lambda) d\left[\sigma(\lambda) - \omega_-(\lambda)\right] = 0,\ r \neq r',\ r_0 < r, r' < \infty\right).$$

Put

$$\begin{aligned} u(r) &= \mathrm{Re}\Gamma_r(0,r), & v(r) &= \mathrm{Im}\Gamma_r(0,r),\ r \geq r_0; \\ \varphi(r,\lambda) &:= \mathrm{Re}\left(e^{-i\frac{1}{2}\lambda r} e(r,\lambda)\right), & \psi(r,\lambda) &:= \mathrm{Im}\left(e^{-i\frac{1}{2}\lambda r} e(r,\lambda)\right), \quad \mathrm{Im}\lambda = 0. \end{aligned}$$

Then $\varphi(r,\lambda)$, $\psi(r,\lambda)$ satisfy the Dirac type system

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi' \\ \varphi' \end{pmatrix} = \frac{\lambda}{2} \begin{pmatrix} \psi \\ \varphi \end{pmatrix} + V(r) \begin{pmatrix} \psi \\ \varphi \end{pmatrix},\ V(r) = \begin{pmatrix} v(r) & u(r) \\ u(r) & -v(r) \end{pmatrix}. \tag{0.1}$$

Let $V_0(r)$ be the potential of the system (0.1) obtained in this way for the non-decreasing function $\sigma(\lambda)$. Considering $V_0(r)$ and $\sigma(\lambda)$ as known we describe explicitly the amendment $V(r) - V_0(r)$ caused by taking $\omega_-(\lambda)$ from $\sigma(\lambda)$.