# On the Uniform Convergence of Pade Approximants for Generalized Nevanlinna Functions

M. Derevyagin
joint work with V.A. Derkach

Let $\varphi$ be given as follows

$$\varphi(\lambda) = r_1(\lambda) \int_a^b \frac{d\mu(t)}{t-\lambda} + r_2(\lambda),$$

where $r_1$, $r_2$ are real rational functions such that $r_1(x) \geq 0$ for $x \in \mathbb{R}$, $r_1(x) = O(1)$ as $x \to +\infty$, and $r_2(x) = o(1)$ as $x \to +\infty$. Then *the n-th diagonal Pade approximant for* $\varphi$ is defined as the rational function $\pi_n(\lambda) = Q_n(\lambda)/P_n(\lambda)$ satisfying the relations

$$\varphi(\lambda) - \pi_n(\lambda) = O(\lambda^{-2n-1}) \ (|\lambda| \to +\infty),$$

$\deg Q_n \leq n$, and $\deg P_n = n$. According to the Pade theorem, there exist n-th diagonal Pade approximants for sufficiently large n. It is proved that *the sequence* $\{\pi_n\}_{n=1}^{\infty}$ *converges to* $\varphi$ *locally uniformly in* $\mathbb{C} \setminus ([a,b] \cup \mathcal{P}(\varphi))$; here $\mathcal{P}(\varphi)$ denotes the set of all poles of $\varphi$. A similar statement for a large class of generalized Nevanlinna functions is proven. The main tool of the proof is the generalized Jacobi matrix associated to $\varphi$, which corresponds to the Schur algorithm for continued fraction expansion of $\varphi$.