On Eigenvalues of Non-Definitizable Differential Operators

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Let $L_{sl} = -\frac{d^2}{dx^2} + q(x)$ be a selfadjoint Sturm-Liouville operator in $L^2(\mathbb{R})$. Let $(Jf)(x) := (\operatorname{sgn} x) f(x)$. The problem under consideration is a detailed description of the spectrum of the J-selfadjoint Sturm-Liouville operator

$$A_{sl} := JL_{sl} = (\operatorname{sgn} x)\left(-\frac{d^2}{dx^2} + q(x)\right).$$

The theory of boundary triples (see, for example, [1]) gives a description of the spectrum $\sigma(A_{sl})$ and the discrete spectrum $\sigma_{disc}(A_{sl})$ in terms of the Weyl function. In this work we consider a question of the existence of eigenvalues in the essential spectrum $\sigma_{ess}(A_{sl})$ of the singular J-selfadjoint Sturm-Liouville operator A_{sl}. Geometric and algebraic multiplicities of eigenvalues will be given. Emphasize that we do not assume that the operator A_{sl} is definitizable.

We apply these results to obtain the following theorems.

Theorem 1 *Let $L_{sl} = -\frac{d^2}{dx^2} + q$ be a Sturm-Liouville operator with a finite-zone potential. Then*

1 *the nonreal spectrum of A_{sl} consists of a finite number of eigenvalues;*

2 *eigenvalues of A_{sl} are isolated and have finite algebraic multiplicity.*

Theorem 2 *There exist a selfadjoint Krein-Feller differential operator $L = -\frac{d^2 f}{dM(x)dx}$ in $L^2(\mathbb{R}, dM(x))$ and a positive constant c such that the operator*

$$A = J(L - c) = (\operatorname{sgn} x)\left(-\frac{d^2 f}{dM(x)dx} - c\right)$$

has an eigenvalue of infinite algebraic multiplicity.

References

[1] V. A. Derkach, M. M. Malamud, *The extension theory of Hermitian operators and the moment problem*, Analiz-3, Itogi nauki i tehn. Ser. Sovrem. mat. i eё pril., V. 5, VINITI, Moscow, 1993 (Russian); English translation: J. Math. Sc.—1995.—V. 73, no. 2.—P.141–242.